

# **Design of reinforced concrete sections according to EN 1992-1-1 and EN 1992-2**

Educational text for practical seminar  
2D members

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## 1. Types of 2D members

### 1.1. Slab

According to EN 1992-1-1, art. 5.3.1(4) a slab is a member, for which the minimum panel dimension is not less than 5 times the overall slab thickness. Slab is loaded only by bending moments and shear forces perpendicular to centroidal plane of slab. Detailing provisions check is performed according to EN 1992-1-1, art. 9.3.

### 1.2. Shell as slab – Shell-slab

The geometry is defined similar to the slab geometry definition. Unlike the slab, the shell-slab can be loaded by bending and membrane actions. Detailing provisions are checked according to rules for slab (EN 1992-1-1, art. 9.3).

### 1.3. Wall

According to EN 1992-1-1, art. 5.3.1(7) a wall is a member, for which following principles are not fulfilled:

- the section depth does not exceed 4 times its width
- the height is at least 3 times the section depth

The wall is loaded only by membrane action and detailing provisions are checked according to EN 1992-1-1, art. 9.6.

### 1.4. Shell as wall – Shell-wall

The geometry is defined similar to the wall geometry definition. Unlike the wall, the shell-wall can be loaded by bending and membrane actions. Detailing provisions are checked according to detailing provisions for wall (EN 1992-1-1, art. 9.6)

### 1.5. Deep beam

According to EN 1992-1-1, art. 5.3.1(3) a deep beam is a member for which the span is less than 3 times the overall section depth. Deep beam can be loaded as the wall only by membrane actions. Detailing provisions are checked according to EN 1992-1-1, art. 9.7.

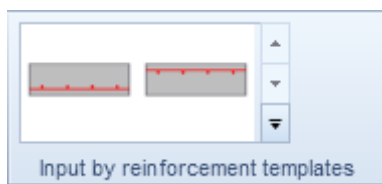
## 2. Input of reinforcement

A cutting with edges 1m x 1m is defined for check. Reinforcement is input to this cutting of 2D member. Reinforcement per linear meter is taken into account for the check of 2D member.

Predefined reinforcement templates can be used to input the reinforcement to the upper and the lower edge. It is possible to input a general reinforcement to the slab cutting.

The reinforcement is defined to the cross-section and to the plan of the 2D cutting.

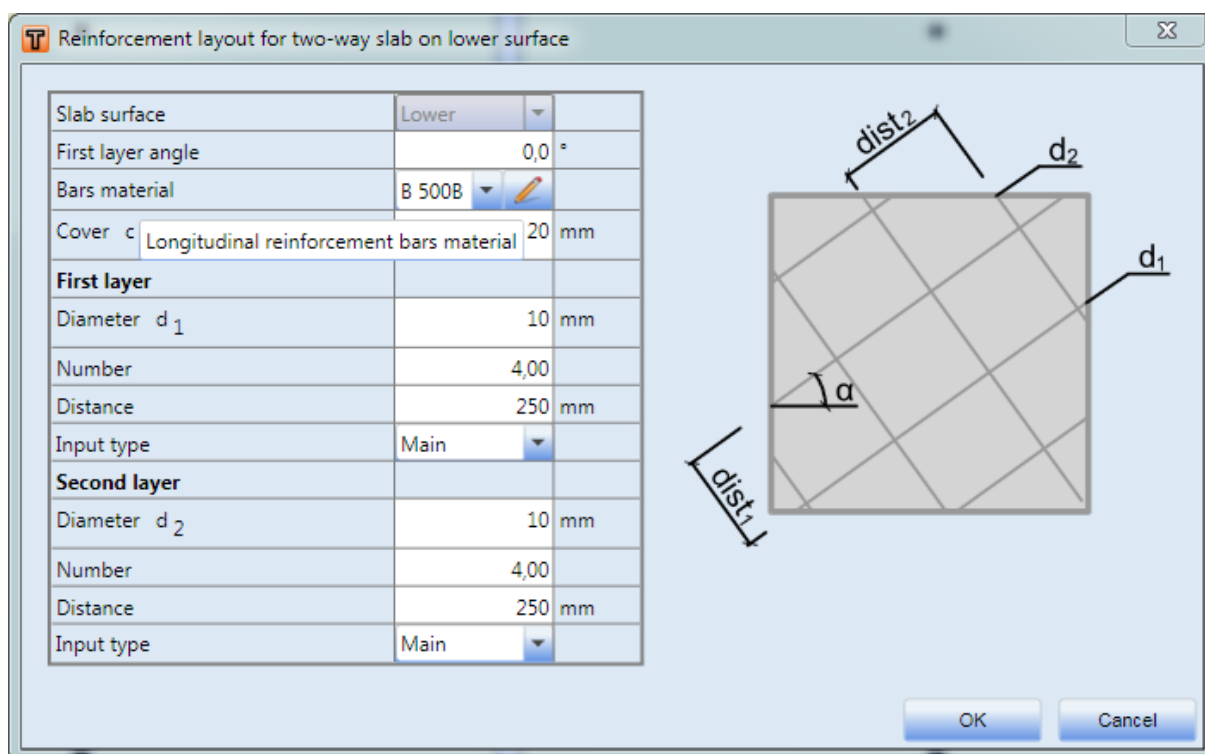
### 2.1. Input of reinforcement using reinforcement templates



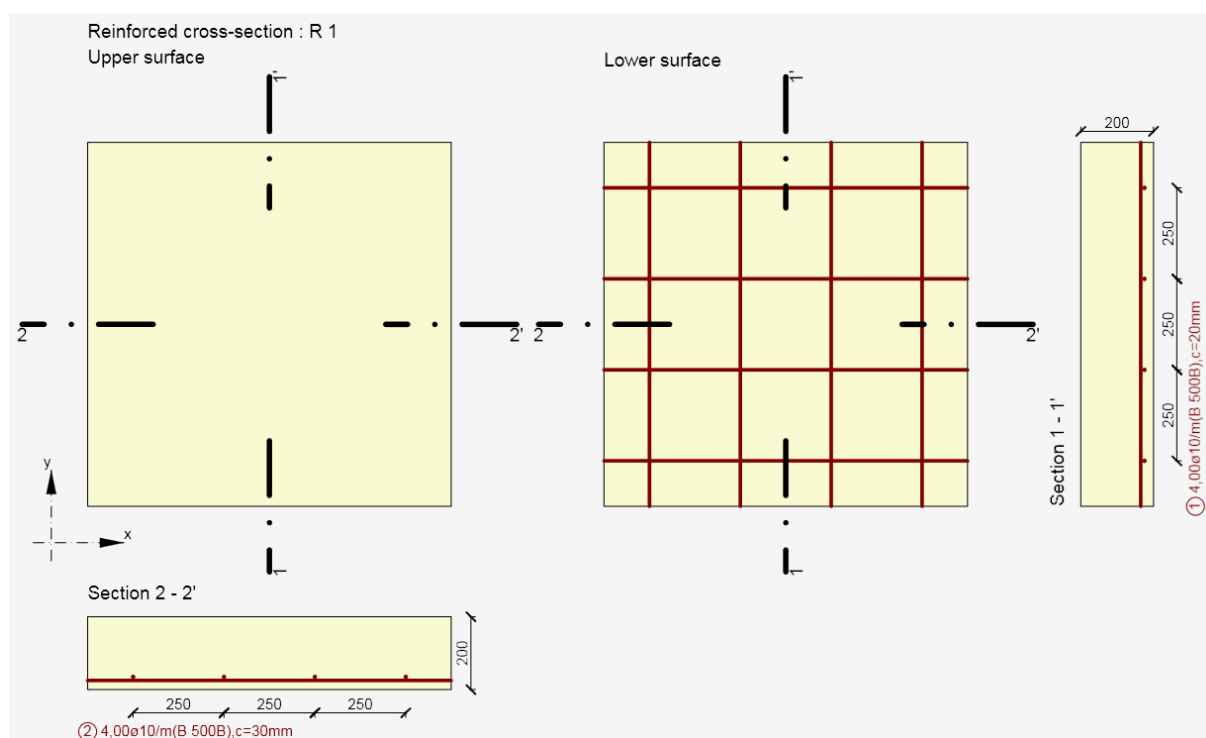
IDEA Concrete supplies two templates for input of reinforcement into 2D element. One template is for input of reinforcement at the upper surface, the other one is for input of reinforcement at the lower surface.

Both templates allow to input orthogonal reinforcement at the one surface of 2D element. Both templates enable to input rotation of reinforcement to the local x-axis of 2D element.

Dialog for definition of 2D reinforcement:



Schema of defined reinforcement at the lower surface of 2D element:



## 2.2. Input of general reinforcement

Input of general reinforcement enables sequential definition of reinforcement into the section and plan of 2D element cutting. Each reinforcement layer is defined in the section and in the plan.

Ø [ mm ]	Number [ /m ]	Distance [ mm ]	Edge bar specification Edge bar distance [ mm ]	Angle [ ° ] As [ mm <sup>2</sup> /m ]	Surface / plane	Type of input	Value [ mm ] Eccentricity [ mm ]	Type	Material
10	4	250	Distance / 2 125	0,0 314	Lower	By clear cover	20 -75	Main	B 500B

## 2.3. Type of reinforcement

Type of reinforcement bars has to be defined to be able to perform the check of detailing provisions. For 2D elements of type

- Slab and shell-slab – for checks according to EN 1992-1-1, art. 9.3.1.1
  - o Main reinforcement
  - o Distribution reinforcement
- Wall, Shell-wall and Deep beam – for check according to EN 1992-1-1, art 9.6.2 and 9.6.3
  - o Horizontal reinforcement
  - o Vertical reinforcement

### Remark:

**The distribution reinforcement** of slabs and shell-slabs is taken into account only for check of detailing provisions, it is not used in other checks of 2D element.

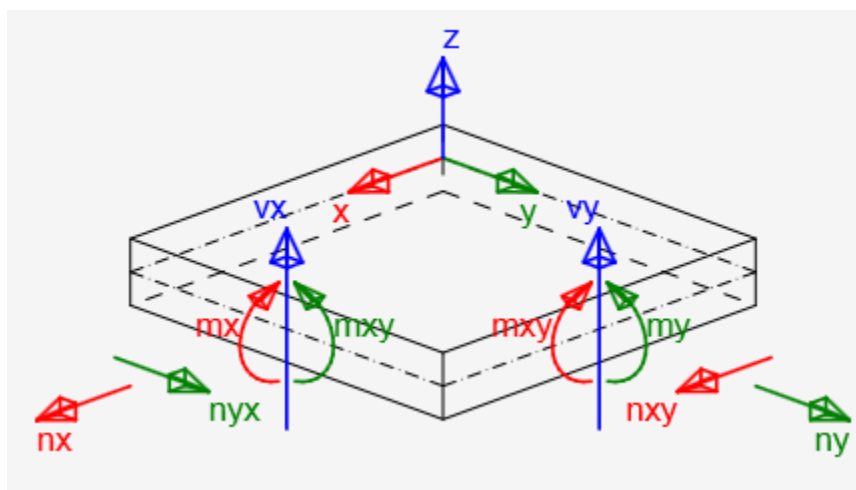
### 3. Internal forces

#### 3.1. Input of internal forces

Input of internal forces of 2D member depends on type of 2D element:

- **Shell-slab** – membrane forces ( $n_x$ ,  $n_y$  and  $n_{xy}$ ), bending moments ( $m_x$ ,  $m_y$  and  $m_{xy}$ ) and shear forces ( $v_x$  and  $v_y$ ) can be entered
- **Shell- wall** – membrane forces ( $n_x$ ,  $n_y$  and  $n_{xy}$ ), bending moments ( $m_x$ ,  $m_y$  and  $m_{xy}$ ) and shear forces ( $v_x$  and  $v_y$ ) can be entered
- **Slab** - only bending moments ( $m_x$ ,  $m_y$  and  $m_{xy}$ ) and shear forces ( $v_x$  and  $v_y$ ) can be entered
- **Wall** – only membrane forces ( $n_x$ ,  $n_y$  and  $n_{xy}$ ) can be entered
- **Deep beam** - only membrane forces ( $n_x$ ,  $n_y$  and  $n_{xy}$ ) can be entered

	Description
$m_{x(y)}$	Bending moment in direction of x (y) axis. Positive value causes tension at the lower surface of 2D element.
$m_{xy(yx)}$	torsional moment about y (x)-axis acting on the edge parallel to the axis x (y). Positive value causes tensional shear stress at the lower surface of 2D element. Because in each point of 2D element theorem on equality of horizontal shear stresses is valid, torsional moments $m_{xy} = m_{yx}$ are equal in each point of 2D element too. Thus only value of $m_{xy}$ is entered in the program.
$n_{x(y)}$	Normal force in direction of x (y)-axis. Positive value acts in direction of x(y) axis and causes tension in section.
$n_{xy(yx)}$	Normal force acting in centre plane in direction of y(x)-axis on edge parallel to x(y) axis. Positive value acts in direction of x(y)-axis. Because in each point of 2D element theorem on equality of horizontal shear stresses is valid, normal forces $n_{xy} = n_{yx}$ are equal in each point of 2D element too. Thus only value of $n_{xy}$ is entered in the program.
$v_{x(y)}$	Shear force acting perpendicular to centre plane on edge parallel to x(y)-axis. Positive value acts in direction of z-axis



Following types of combinations has to be defined for checks:

- **Ultimate limit state** – internal forces components defined for this type of combinations are used for ULS checks of 2D elements:
  - Capacity N-M-M
  - Response N-M-M
  - Interaction
 and the check of detailing provisions
- **Characteristic** – internal forces components defined for this type of combinations are used for check of stress limitation (SLS)
- **Quasi-permanent** – internal forces components defined for this type of combinations are used for check of crack width (SLS)

Combination type	$m_x$ [kNm/m]	$m_y$ [kNm/m]	$m_{xy}$ [kNm/m]	$n_x$ [kN/m]	$n_y$ [kN/m]	$n_{xy}$ [kN/m]	$v_x$ [kN/m]	$v_y$ [kN/m]
Fundamental ULS	1,40	1,17	-2,25	0,00	0,00	0,00	4,97	1,93
Characteristic	1,01	0,85	-1,63	0,00	0,00	0,00	0,00	0,00
Quasi-permanent	1,01	0,85	-1,63	0,00	0,00	0,00	0,00	0,00

#### Remark:

Internal forces components  $v_x$  and  $v_y$  are not required to be entered for combination types **Characteristic** and **Quasi-permanent**, because those values are not used in checks.

### 3.2. Determining direction of check

Direction of check has to be determined for proper check of 2D element. Direction of check can be entered for each combination type separately, using following two methods:

- **User defined direction** – user defines check direction as an angle to x-axis in plane of 2D element. This option is set as default for combination type ULS and the predefined value of angle is 0 degrees. Checks are performed in following directions:
  - Defined direction
  - Direction perpendicular to defined direction
  - Direction of compression diagonal at the upper surface
  - Direction of compression diagonal at the lower surface
- **Direction of principal stresses** – check direction is calculated automatically as a direction of principal stresses at the upper and at the lower surface of 2D element. This option is set as default for combination types Characteristic and Quasi-permanent. Checks are performed in following directions:
  - Direction of principal stresses at the lower surface
  - Direction perpendicular to the direction of principal stresses at the lower surface
  - Direction of compression diagonal at the lower surface
  - Direction of principal stresses at the upper surface
  - Direction perpendicular to the direction of principal stresses at the upper surface
  - Direction of compression diagonal at the upper surface

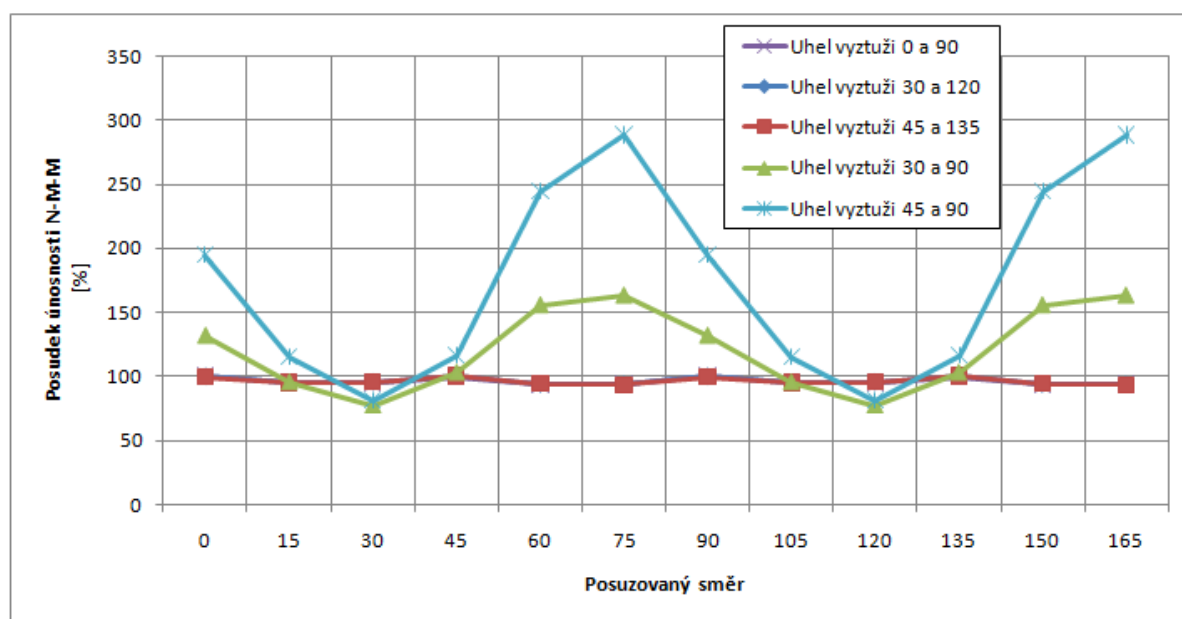


Combination type	Checked direction	Angle from x-axis [°]
Fundamental ULS	User direction	0,0
Characteristic	Principal stress direction	
Quasi-permanent	Principal stress direction	

### 3.2.1. Analysis of check direction for ultimate limit state

#### Analysis 1

For 2D element loaded only by bending moments ( $m_x = 20 \text{ kNm/m}$ ,  $m_y = 10 \text{ kNm/m}$ ,  $m_{xy} = 5 \text{ kNm/m}$ ) angle of reinforcement and angle of check direction were changed for ultimate limit state. Results are displayed in following graph:

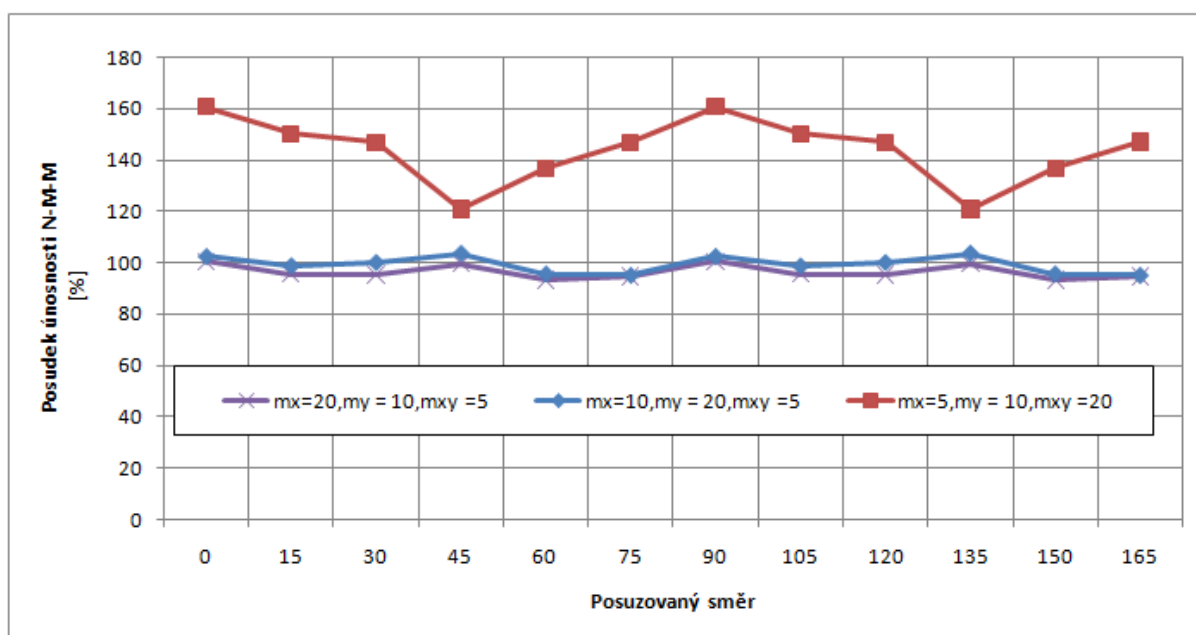


The analysis implies:

- If reinforcement bars are perpendicular to each other, check results are similar for different check direction angles, are not dependent on defined reinforcement angle and the maximal value of check is found for angles 0, 45 and 90 degrees. Thus check can be performed for predefined direction of check 0 degrees.
- If reinforcement bars are not perpendicular to each other, results of checks are different significantly and the maximal check value is achieved approximately in the direction corresponding with the direction of average reinforcement. Thus it is recommended to change predefined check direction or to perform checks in more directions in cases, when reinforcement bars are not perpendicular to each other.

#### Analysis 2

For 2D element reinforced with rectangular reinforcement values of bending moments and angles for check were changed for ULS. Results are displayed in following graph:



The analysis implies that even for different values of bending moments the maximal value of ultimate limit state check is found for check directions 0, 45 and 90 degrees. Thus the check can be performed for predefined check angle 0 degrees. Similar conclusion is valid for 2D elements loaded by normal force only or loaded by normal force combined with bending moments.

### 3.3. Recalculation of internal forces to directions of check

Defined internal forces are recalculated to the check directions using Baumann transformation formula, described e.g. in [3]. The calculation procedure is as follows:

1. Calculation of normal forces at both surfaces of 2D element
2. Calculation of principal forces at both surfaces of 2D element
3. Calculation of recalculated forces for each surface to the defined direction of check
4. Calculation of recalculated forces for each surface to the centre
5. Recalculation of shear forces to the defined direction of check

#### 3.3.1. Calculation of normal forces at both surfaces of 2D element

Defined internal forces are recalculated to both surfaces using following formulas:

$$n_{x,low(upp)} = \frac{n_x}{2} + (-) \frac{m_x}{z}$$

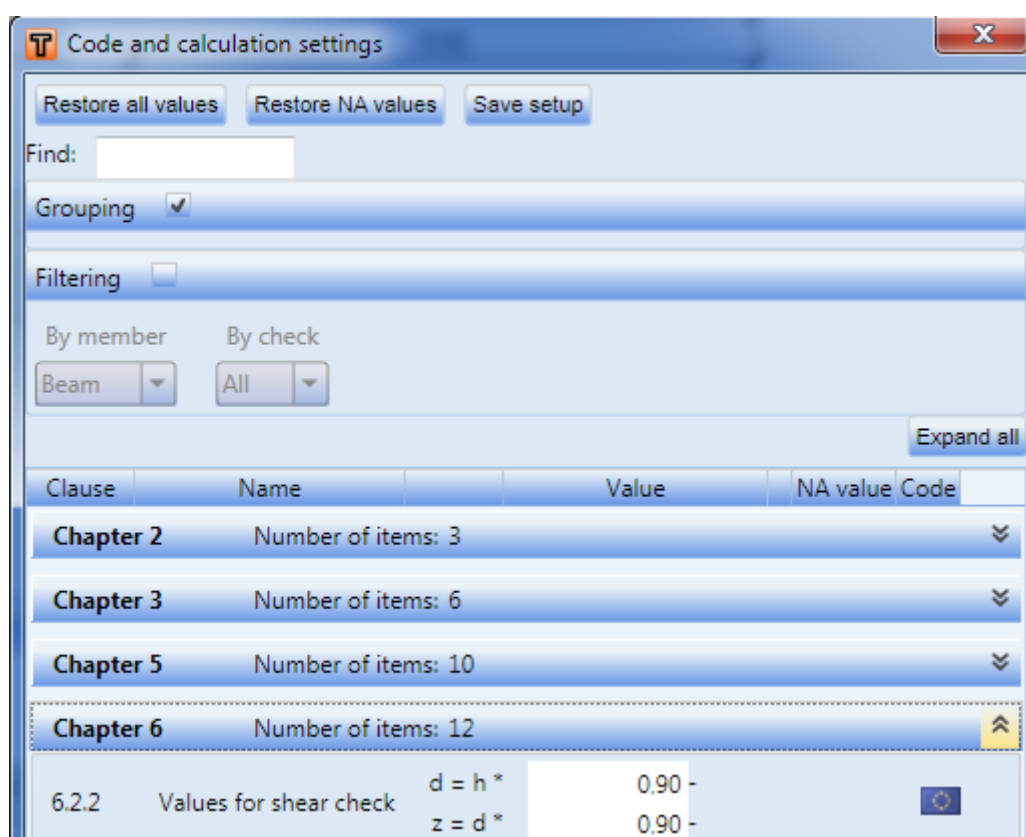
$$n_{y,low(upp)} = \frac{n_y}{2} + (-) \frac{m_y}{z}$$

$$n_{xy,low(upp)} = \frac{n_{xy}}{2} + (-) \frac{m_{xy}}{z}$$

The arm of internal forces ( $z$ ) has to be determined for the recalculation of internal forces. The arm of internal forces is determined from method of limit strain at loading by principal bending moment in directions of principal moments  $m_1$  at both surfaces. If principal moments are equal zero or if the equilibrium is not found in direction of principal moments, the arm of internal forces is determined according to formula

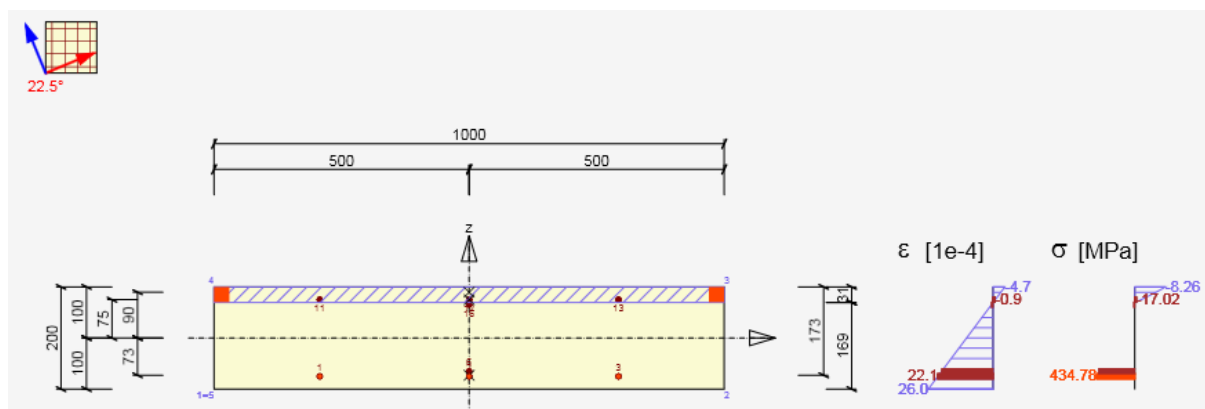
$$z = x \cdot d,$$

	Description
x	Coefficient for calculation of internal forces arm, defined in National code setup.
d	Effective height of cross-section calculated separately for the upper and the lower surfaces of 2D element. For the lower surface it is a distance from centroid of reinforcement bars at the lower surface to the upper edge of cross-section. For the upper surface it is a distance from centroid of reinforcement bars at the upper surface to the lower edge of cross-section.

**Remark:**

The arm of internal forces can be verified in the Response N-M-M check. Only the bending moments have to be entered and the check direction has to correspond with the direction of principal moment.

In the following picture a verification of internal forces arm is displayed for bending moments  $m_x = 20 \text{ kNm/m}$ ,  $m_y = 10 \text{ kNm/m}$ ,  $m_{xy} = 5 \text{ kNm/m}$ . The direction of principal moments has been calculated  $\alpha_{m1} = 22,5 \text{ degrees}$  and the response of cross-section was calculated to determine the internal forces arm.



Plane of Strain

x [ mm ]	d [ mm ]	z [ mm ]	$\epsilon_x$ [ 1e-4 ]	$\epsilon_y$ [ 1e-4 ]	$\epsilon_z$ [ 1e-4 ]
31	173	163	10,6	0,0	-153,2

**Remark:**

Internal forces arms for recalculation of internal forces to the direction of check and internal forces arms for checks can be different, because internal forces arm for recalculation is determined for cross-section loaded by principal moment in direction of principal moments, and the internal forces arm for check is determined for cross-section loaded by bending moments and normal forces in the direction of check. Values of internal forces arms for all combination types are displayed in the table **Recalculated forces** in the navigator **Internal forces in section**.

## Recalculated forces

Normal forces (design and in principal stress direction) at surfaces for ULS combination

Surface	Checked direction	n1 [ kN/m ]	n2 [ kN/m ]	$\alpha_{n1}$ [ ° ]	z [ mm ]	Angle [ ° ]	n surface [ kN/m ]
Upper	Principal stress direction	-48,77	-135,76	-67,5	163	-67,5	-48,77
Upper	Perpendicular direction	-48,77	-135,76	-67,5	163	22,5	-135,76
Upper	Compressive concrete strut	-48,77	-135,76	-67,5	163	67,5	0,00
Lower	Principal stress direction	135,76	48,77	22,5	163	22,5	135,76
Lower	Perpendicular direction	135,76	48,77	22,5	163	112,5	48,77

**3.3.2. Calculation of internal forces at both surfaces**

Principal forces at both surfaces of 2D element are calculated using formula

$$n_{1,bot(top)} = \frac{n_{x,low(upp)} + n_{y,low(upp)}}{2} + \frac{1}{2} \sqrt{(n_{x,low(upp)} - n_{y,low(upp)})^2 + 4 \cdot n_{xy,low(upp)}}$$

$$n_{2,bot(top)} = \frac{n_{x,low(upp)} + n_{y,low(upp)}}{2} - \frac{1}{2} \sqrt{(n_{x,low(upp)} - n_{y,low(upp)})^2 + 4 \cdot n_{xy,low(upp)}}$$

And the direction of principal forces is calculated using formula

$$\alpha_{n1,low(upp)} = 0,5 \cdot \tan^{-1} \left( \frac{2 \cdot n_{xy,low(upp)}}{n_{x,low(upp)} - n_{y,low(upp)}} \right)$$

**Remark:**

Principal forces and the direction of principal forces for both surfaces of 2D element are displayed for all combination types in the table **Recalculated forces** in the navigator **Internal forces in section**.

Normal forces (design and in principal stress direction) at surfaces for ULS combination

Surface	Checked direction	n1 [ kN/m ]	n2 [ kN/m ]	$\alpha_{n1}$ [ ° ]	z [ mm ]	Angle [ ° ]	n surface [ kN/m ]
Upper	User direction	-48,77	-135,76	-67,5	163	22,5	-135,76
Upper	Perpendicular direction	-48,77	-135,76	-67,5	163	112,5	-48,77
Upper	Compressive concrete strut	-48,77	-135,76	-67,5	163	157,5	0,00
Lower	User direction	135,76	48,77	22,5	163	22,5	135,76
Lower	Perpendicular direction	135,76	48,77	22,5	163	112,5	48,77
Lower	Compressive concrete strut	135,76	48,77	22,5	163	67,5	0,00

### 3.3.3. Calculation of recalculated internal forces at surfaces to the defined check direction

Recalculation of principal forces to the check directions is performed separately for each surface using Baumann transformation formula

$$n_{surface,i,low(upp)} = \frac{n_{1,low(upp)} \cdot \sin(\alpha_{j,low(upp)}) \cdot \sin(\alpha_{k,low(upp)}) + n_{2,low(upp)} \cdot \cos(\alpha_{j,low(upp)}) \cdot \cos(\alpha_{k,low(upp)})}{\sin(\alpha_{j,low(upp)} - \alpha_{i,low(upp)}) \cdot \sin(\alpha_{k,low(upp)} - \alpha_{i,low(upp)})}$$

	Description
i, j, k, i	Index of check direction (internal forces recalculation direction) i, j, k, i = 1,2,3,1 . E. G. For lower surface and calculation of force in j-direction (angle $\alpha_2$ ) the formula is: $n_{surface,2,low} = \frac{n_{1,low} \cdot \sin \alpha_{3,low} \cdot \sin \alpha_{1,low} + n_{2,low} \cdot \cos \alpha_{3,low} \cdot \cos \alpha_{1,low}}{\sin(\alpha_{3,low} - \alpha_{2,low}) \cdot \sin(\alpha_{1,low} - \alpha_{2,low})}$
$\alpha_{i,j,k,low(upp)}$	Angle between defined check direction or the direction of compressive strut and the direction of principal forces at the lower or upper surface of 2D element. <ul style="list-style-type: none"> <li>Defined check direction <math>\alpha_{1,low(upp)} = \alpha_1 - \alpha_{low(upp)}</math></li> <li>Dir. perpendicular to the defined direction <math>\alpha_{2,low(upp)} = \alpha_2 - \alpha_{low(upp)}</math></li> <li>Direction of check for compressive strut <math>\alpha_{3,low(upp)} = \alpha_3 - \alpha_{low(upp)}</math></li> </ul>
$\alpha_1$	Defined check direction for particular combination
$\alpha_2$	Direction perpendicular to defined direction, $\alpha_2 = \alpha_1 + 90$ degrees
$\alpha_3$	Check direction in the direction of compressive strut in the plane of 2D element. This direction is optimized to minimize the force in this direction.

**Remark:**

- If the **check direction** is identical with **Principal stresses direction**, the forces in compressive strut are zero, thus this direction is neglected in check
- The direction of compressive strut for all states of stress except the hyperbolic state of stress ( $n_{1,low(upp)} > 0$  and  $n_{1,low(upp)} < 0$ ) can be calculated according to formula:  

$$\alpha_3 = 0,5(\alpha_1 + \alpha_2)$$
- Recalculated internal forces for both surfaces of 2D element a and all check directions including the direction of compressive strut are displayed in the table **Recalculated forces**

**Recalculated forces**

Normal forces (design and in principal stress direction) at surfaces for ULS combination

Surface	Checked direction	n1 [ kN/m ]	n2 [ kN/m ]	$\alpha$ n1 [ ° ]	z [ mm ]	Angle [ ° ]	n surface [ kN/m ]
Upper	User direction	-48,77	-135,76	-67,5	163	0,0	-92,27
Upper	Perpendicular direction	-48,77	-135,76	-67,5	163	90,0	-30,76
Upper	Compressive concrete strut	-48,77	-135,76	-67,5	163	45,0	-61,51
Lower	User direction	135,76	48,77	22,5	163	0,0	153,78
Lower	Perpendicular direction	135,76	48,77	22,5	163	90,0	92,27
Lower	Compressive concrete strut	135,76	48,77	22,5	163	135,0	-61,51

**3.3.4. Transformation of recalculated internal forces to the centroid of cross-section**

For the check of 2D element the surface forces in particular direction have to be recalculated to the centroid of cross-section. The result is normal force  $n_{d,i}$  and bending moment  $m_{d,i}$  acting in the centroid of the 2D element cross-section.

$$m_{d,i} = n_{lower,i} \cdot z_{s,low} + n_{upper,i} \cdot z_{s,upp}$$

$$n_{d,i} = n_{lower,i} + n_{upper,i}$$

	Description
$n_{lower,i}$	Recalculated surface forces at lower surface in the $i^{th}$ check direction, when $n_{lower,i} = n_{surface,low,i}$ .
$n_{upper,i}$	Recalculated internal forces at upper surface in the $i^{th}$ check direction, when $n_{upper,i} = n_{surface,upp,i}$ .
$z_{s,low(upp)}$	Distance of centroid of compressed concrete or centroid of reinforcement at the lower (upper) surface, when $z = z_{s,low} + z_{s,upp}$

**Remark:**

If the directions of compressive struts at the lower and the upper surface are different, for the recalculation of forces to the centroid it is necessary to calculate virtual forces at the bottom surface in the direction of compressive strut at the upper surface and vice versa.

## Recalculated design forces

Design forces in centroidal plane for ULS combination

Angle [°]	Concrete strut	n upper [kN/m]	n lower [kN/m]	n d [kN/m]	m d [kNm/m]	v d [kN/m]
0,0	No	-92,27	153,78	61,51	19,43	0,00
45,0	Yes	-61,51	61,51	0,00	10,00	0,00
90,0	No	-30,76	92,27	61,51	9,43	0,00
135,0	Yes	61,51	-61,51	0,00	-10,00	0,00

### Recalculated forces

Normal forces (design and in principal stress direction) at surfaces for ULS combination

Surface	Checked direction	n1 [kN/m]	n2 [kN/m]	α n1 [°]	z [mm]	Angle [°]	n surface [kN/m]
Upper	User direction	-48,77	-135,76	-67,5	163	0,0	-92,27
Upper	Perpendicular direction	-48,77	-135,76	-67,5	163	90,0	-30,76
Upper	Compressive concrete strut	-48,77	-135,76	-67,5	163	45,0	-61,51
Lower	User direction	135,76	48,77	22,5	163	0,0	153,78
Lower	Perpendicular direction	135,76	48,77	22,5	163	90,0	92,27
Lower	Compressive concrete strut	135,76	48,77	22,5	163	135,0	-61,51

Virtual force at the upper surface in the direction of the compressive strut at the bottom surface (angle 135 degrees)

Virtual force at the bottom surface in the direction of compressive strut at the upper surface (angle 45 degrees)

### 3.3.5. Recalculation of shear forces to the defined direction of check

Shear forces are recalculated to the direction of check using formula

$$v_{d,i} = v_x \cdot \cos(\alpha_i) + v_y \cdot \sin(\alpha_i)$$

and the maximal shear force is

$$v_{d,max} = \sqrt{v_x^2 + v_y^2}$$

and it acts in direction

$$\beta = \tan^{-1} \left( \frac{v_y}{v_x} \right)$$

	Description
$\alpha_i$	Check angle in the $i^{\text{th}}$ direction

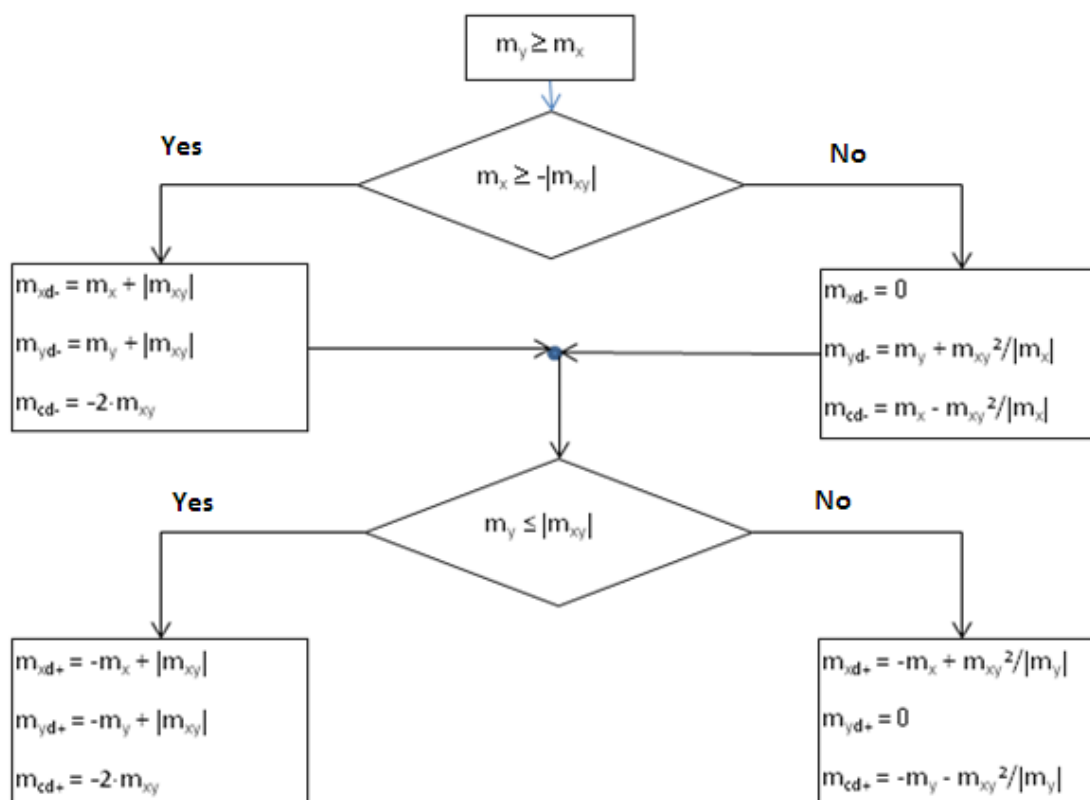
#### Remark:

When checking 2D element with relatively large shear forces it is suitable to check the 2D element in the direction of maximal shear force, it means that the defined direction check corresponds to angle  $\beta$

### 3.4. Comparison of internal forces recalculation using various methods

#### 3.4.1. Recalculation of forces acc. to ENV 1992-1-1

Method described in ENV 1992-1-1 is used in several programs and in praxis to calculate dimensioning forces. In general this method corresponds to method in ČSN 73 1204, but ENV takes into account only perpendicular reinforcement directions. Calculation of dimensioning forces with influence of torsional moment is described in following flowchart, where  $m_y \geq m_x$ . Similar diagram can be created for moments  $m_y < m_x$



	Description
$m_{xd+}, m_{xd-}$	Dimensioning bending moment in the x-axis direction for design and check of reinforcement at the lower (-) or the upper (+) surface
$m_{yd+}, m_{yd-}$	Dimensioning bending moment in the y-axis direction for design and check of reinforcement at the lower (-) or the upper (+) surface
$m_{cd+}, m_{cd-}$	Dimensioning bending moment in the compressive concrete strut at the lower (-) or the upper (+) surface, which has to be carried by concrete

Values of recalculated dimensioning forces for type of member = Slab, calculated using method described in ENV, are displayed in the following table:



Index	$m_x$ [kNm/m]	$m_y$ [kNm/m]	$m_{xy}$ [kNm/m]	$m_{xd-}$ [kNm/m]	$m_{yd-}$ [kNm/m]	$m_{cd-}$ [kNm/m]	$m_{xd+}$ [kNm/m]	$m_{yd+}$ [kNm/m]	$m_{cd+}$ [kNm/m]
1	-2.93	-2.93	-1.95	-1.63	0	-4.23	4.88	4.88	-3.91
2	0.2	-7.14	-2.31	0.95	0	-7.89	2.11	9.45	-4.62
3	-1.11	-10.14	-0.31	-1.1	0	-10.15	1.42	10.45	-0.63
4	-7.14	0.2	-2.31	0	0.95	-7.89	9.45	2.11	-4.62
5	7.26	7.26	-2.03	9.29	9.29	-4.05	0	-6.7	-7.83
6	5.6	11.99	1.46	7.05	13.45	-2.91	-5.42	0	-12.17
7	-10.14	-1.11	-0.31	0	-1.1	-10.15	10.45	1.42	-0.63
8	11.99	5.6	1.46	13.45	7.05	-2.91	0	-5.42	-12.17
9	9.63	9.63	6.4	16.02	16.02	-12.79	-5.38	0	-13.87

In IDEA Concrete values of moments at upper and lower surface are not displayed, but values of normal forces at both surfaces and values of moments recalculated to the centroid of the cross-section.

## Recalculated design forces

Design forces in centroidal plane for ULS combination

Angle [ ° ]	Concrete strut	n upper [ kN/m ]	n lower [ kN/m ]	n d [ kN/m ]	m d [ kNm/m ]	v d [ kN/m ]
0.0	No	30.12	-6.05	24.07	-2.79	0.00
45.0	Yes	24.07	-24.07	0.00	-3.90	0.00
90.0	No	30.12	-6.05	24.07	-2.79	0.00
135.0	Yes	-24.07	24.07	0.00	3.90	0.00

## Recalculated forces

Normal forces (design and in principal stress direction) at surfaces for ULS combination

Surface	Checked direction	n1 [ kN/m ]	n2 [ kN/m ]	$\alpha_{n1}$ [ ° ]	z [ mm ]	Angle [ ° ]	n surface [ kN/m ]
Upper	User direction	30.12	6.05	45.0	162	0.0	30.12
Upper	Perpendicular direction	30.12	6.05	45.0	162	90.0	30.12
Upper	Compressive concrete strut	30.12	6.05	45.0	162	135.0	-24.07
Lower	User direction	-6.05	-30.12	-45.0	162	0.0	-6.05
Lower	Perpendicular direction	-6.05	-30.12	-45.0	162	90.0	-6.05
Lower	Compressive concrete strut	-6.05	-30.12	-45.0	162	45.0	-24.07

Moments at lower and upper surface can be calculated using surface forces, which are displayed in numerical output, using formula:

$$m_{surface,i,low(upp)} = n_{surface,i,low(upp)} \cdot z$$

Values of surface forces and recalculated moments are displayed in following tables:

Index	$n_{surface,1,low}$ [kN/m]	$n_{surface,2,low}$ [kN/m]	$n_{surface,3,low}$ [kN/m]	$n_{surface,1,upp}$ [kN/m]	$n_{surface,2,upp}$ [kN/m]	$n_{surface,3,upp}$ [kN/m]
1	-6.05	-6.05	-24.07	30.12	30.12	-24.07
2	15.49	-29.81	-28.52	13.02	58.33	-28.52
3	-4.94	-60.68	-3.83	8.77	64.51	-3.83
4	-29.81	15.49	-28.52	58.33	13.02	-28.52
5	57.35	57.35	-25.06	-32.28	-32.28	-25.06
6	43.58	83.02	-18.02	-25.56	-65.00	-18.02
7	-60.68	-4.94	-3.83	64.51	8.77	-3.83
8	83.02	43.58	-18.02	-65.00	-25.56	-18.02
9	98.95	98.95	-79.01	-19.94	-19.94	-79.01

Index	$m_{surface,1,low}$	$m_{surface,2,low}$	$m_{surface,3,low}$	$m_{surface,1,upp}$	$m_{surface,2,upp}$	$m_{surface,3,upp}$
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	( $m_{xd-}$ ) [kNm/m]	( $m_{yd-}$ ) [kNm/m]	( $m_{cd-}$ ) [kNm/m]	( $m_{xd+}$ ) [kNm/m]	( $m_{yd+}$ ) [kNm/m]	( $m_{cd+}$ ) [kNm/m]
1	-0.98	-0.98	-3.90	4.88	4.88	-3.90
2	2.51	-4.83	-4.62	2.11	9.45	-4.62
3	-0.80	-9.83	-0.62	1.42	10.45	-0.62
4	-4.83	2.51	-4.62	9.45	2.11	-4.62
5	9.29	9.29	-4.06	-5.23	-5.23	-4.06
6	7.06	13.45	-2.92	-4.14	-10.53	-2.92
7	-9.83	-0.80	-0.62	10.45	1.42	-0.62
8	13.45	7.06	-2.92	-10.53	-4.14	-2.92
9	16.03	16.03	-12.80	-3.23	-3.23	-12.80

The table shows that moments at slab surfaces calculated in IDEA Concrete and calculated according to method described in ENV, correspond only at one surface. This difference is caused by different optimisation of concrete strut. Method used in IDEA Concrete searches for the angle of compressive strut at the minimal force in the strut. The method described in ENV searches for minimal sum of negative forces from all directions.

### 3.4.2. Comparison of internal forces calculation with programs RFEM and SCIA Engineer

To compare results of recalculated internal forces in programs IDEA Concrete, RFEM and SCIA Engineer (SEN) a simple model of slab of dimensions 6 m x 4 m and thickness 200mm was prepared. Slab is supported with line support at edges and loaded with uniform load 10 kN/m<sup>2</sup>.

To simplify the presentation only values of recalculated internal forces in one longitudinal section are displayed. The section distance from slab edge is 1.5m. The internal forces calculated in program RFEM were used as input values for IDEA Concrete.

	$m_{x,lower}$ [kNm/m]			$m_{y,lower}$ [kNm/m]			$m_{c,lower}$ [kNm/m]		
Position [m]	RFEM	SEN	IDEA	RFEM	SEN	IDEA	RFEM	SEN	IDEA
0,00	1,794	1,794	1,794	1,129	1,130	1,131	-3,223	-3,222	-3,227
0,50	6,281	6,282	6,297	6,398	6,398	6,414	-5,378	-5,378	-5,391
1,00	7,349	7,349	7,358	8,911	8,912	8,924	-4,163	-4,164	-4,168
1,50	7,620	7,620	7,623	10,758	10,759	10,763	-3,318	-3,318	-3,320
2,00	7,372	7,373	7,384	11,842	11,843	11,862	-2,221	-2,221	-2,224
2,50	6,908	6,908	6,896	12,245	12,246	12,225	-1,109	-1,109	-1,106
3,00	6,371	6,371	6,366	12,007	12,008	11,999	0,000	0,000	0,000
3,50	6,908	6,908	6,896	12,245	12,246	12,225	-1,109	-1,109	-1,106
4,00	7,372	7,373	7,384	11,842	11,843	11,862	-2,221	-2,221	-2,224
4,50	7,620	7,620	7,623	10,758	10,759	10,763	-3,318	-3,318	-3,320
5,00	7,349	7,349	7,358	8,911	8,912	8,924	-4,163	-4,164	-4,168
5,50	6,281	6,282	6,297	6,398	6,398	6,414	-5,378	-5,378	-5,391
6,00	1,794	1,794	1,794	1,129	1,130	1,131	-3,223	-3,222	-3,227

The table shows good compliance of forces calculated in particular programs.

## 4. Check

As described in 3.3.4 Transformation of recalculated internal forces to the centroid of cross-section, surface dimensioning forces are transformed to the centroid of 2D element cross-section. The result of this transformation is bending moment and normal force, acting in the centroid of rectangular cross-section, which edge length is 1m and height corresponds to the thickness of the slab.

The checks of 2D element are performed in all defined directions at once. The program automatically converts the reinforcement to the check direction using formula

$$A_{s_{i,\alpha}} = A_s \cdot \cos^2(\alpha_i)$$

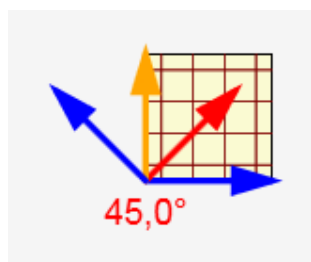
	Description
$A_{s_{i,\alpha}}$	The area of the $i^{\text{th}}$ reinforcement layer recalculated to the direction $\alpha$
$A_s$	The area of the $i^{\text{th}}$ reinforcement layer of 2D element
$\alpha_i$	The angle between $i^{\text{th}}$ reinforcement layer and the check direction

### Remark:

**Distributive reinforcement** in 2D elements of type slab and shell-slab is taken into account only in detailing provision check, it is not used in other 2D element checks.

### 4.1. Results of check in defined directions

All enabled checks are performed in all required directions automatically. The presentation of results is similar to presentation of 1D element results. The presentation for 2D elements enables to set direction to be presented. Results for 2D elements are presented in check directions. All directions, in which the checks were calculated, are drawn in graphical presentation.



The arrows in the picture represent check directions, where the orange is the direction of maximal check value and the red is the current check direction. To change the current direction click the arrow or click the appropriate button in the ribbon.

### Remark:

After the calculation finishes, the check directions in all checks are set to the direction of maximal cross-section utilisation.

Results in particular checks are presented in the current direction. The angle of check is displayed above the table with the check summary.

The results in the extreme direction are printed in report.

## 4.2. Ultimate limit state

The principles of ULS checks are described in theoretical background manual for 1D elements. Only the differences for 2D elements are described in following chapters.

### 4.2.1. Capacity check

The capacity check does not differ from 1D elements checks. The load acts only in one plane, thus the check type is N + M.

### 4.2.2. Response check

The response checks for particular check directions use the same algorithms as checks of 1D elements.

### 4.2.3. Interaction check

Unlike for 1D elements, the interaction check is performed only to evaluate the exploitation V + M, the interaction of shear and bending moment. The separate shear check is not performed in current version. Values  $V_{Rd,c}$  and  $V_{Rd,max}$  can be verified in summary table of the interaction check.

N Ed [ kN ]	M Edy [ kNm ]	V Ed [ kN ]	V Rd,c [ kN ]	V Rd,max [ kN ]	Value V+M [ % ]	Value [ % ]	Limit [ % ]	Check
61,51	19,43	0,00	63,14	622,44	100,00	100,00	100,00	OK

### 4.2.4. Capacity check comparison between IDEA Concrete, RFEM and SCIA Engineer

To compare the capacity check results with RFEM and SCIA Engineer the same data as described in chapter 3.4.2 Comparison of internal forces calculation with programs RFEM and SCIA Engineer were used. The comparison was done in two points of slab.

Because programs RFEM and SEN do not check the real reinforcement in the slab, but only design the necessary reinforcement area, two methods were used to compare the calculation. The first one compares the exploitation of cross-section for required reinforcement designed in RFEM and SEN, assuming that the cross-section is exploited to 100% just when using the calculated required area of reinforcement.

The exploitation of cross-section reinforced in IDEA concrete can be expressed relatively then.

$$\text{Relative exploitation} = A_{s, \text{req}} / A_{s, \text{RCS}} \cdot 100 [\%]$$

	Description
$A_{s, \text{req}}$	Required reinforcement area calculated in RFEM or SEN
$A_{s, \text{RCS}}$	Area of reinforcement in IDEA Concrete
100 [%]	Percentage

The cross-section in IDEA Concrete was reinforced at the lower surface using reinforcement  $\phi 10$  in 200 mm distances in both directions, the area of reinforcement in both directions is 314 mm<sup>2</sup>.

Layer	$\phi$ [mm]	n [mm]	Distance [mm]	Type first bar position	First bar position [mm]	Angle [°]	As [mm <sup>2</sup> ]	Surface for cover	Position type	Value [mm]	Type	Material
1	10	4,00	250	Distance / 2	125	0,0	314	Lower	By clear cover	20	Main	B 500A
2	10	4,00	250	Distance / 2	125	90,0	314	Lower	By clear distance from layer 1	0	Main	B 500A

Comparison results are in the following table:

Position		Direction	Reinforcement area As [mm <sup>2</sup> ]			Cross-section exploitation [%]			Relative exploitation [%]	
			RFEM	SEN	IDEA	RFEM	SEN	IDEA	RFEM	SEN
0,5	0,5	x lower	116	123	314	100	100	37	37	39
0,5	0,5	y lower	129	137	314	100	100	39	41	44
3,0	2,0	x lower	87	93	314	100	100	29	28	30
3,0	2,0	y lower	174	183	314	100	100	58	55	58

The table shows good compliance of exploitation for all programs.

The reinforcement with the approximately same area was defined in IDEA Concrete as calculated required reinforcement in RFEM and SEN for the second method. Afterwards the exploitation of cross-section was compared. Results are displayed in following table:

Position		Direction	Reinforcement area As [mm <sup>2</sup> ]			Cross-section exploitation [%]		
			RFEM	SEN	IDEA	RFEM	SEN	IDEA
0,5	0,5	x lower	116	123	$\phi 10 / 650\text{mm}^2$	123	100	100
0,5	0,5	y lower	129	137	$\phi 10 / 600\text{mm}^2$	137	100	94
3	2	x lower	87	93	$\phi 10 / 800\text{mm}^2$	93	100	103
3	2	y lower	174	183	$\phi 10 / 450\text{mm}^2$	183	100	93

The good compliance of results is here too.

### 4.3. Serviceability limit state

#### 4.3.1. Stress limitation

Check of stress limitation does not differ from checks for 1D elements.

#### 4.3.2. Crack width check

In advance to 1D elements check the direction of crack can be drawn for 2D elements.

### 4.4. Detailing provisions

Detailing provisions check of 2D elements can be split into two basic groups:

- Reinforcement percentage check
- Bar distances check

The detailing provisions check depends on type of 2D element too. The separate checks for the main and for the distribution reinforcement are performed for shell-slab and slab elements. The vertical and horizontal reinforcement is distinguished for wall elements.

The reinforcement percentage check is performed in the direction of principal stresses. The reinforcement defined in cutting of 2D element (except the distribution reinforcement) is transformed to the principal stresses directions.

The bars distance check is performed perpendicular to the direction of defined reinforcement. This check is performed for all defined reinforcement layers and the limit values depend on the type of checked element and on the type of defined reinforcement.

Check of detailing provisions of longitudinal reinforcement

Type	Value calc	Value lim	Ratio [ % ]	Check
Minimal reinf. ratio for main reinforcement (9.3.1.1 (1)) [%]	0,19	0,15	79,11	OK
Maximal reinf. ratio for main reinforcement (9.3.1.1 (1)) [%]	0,16	4,00	3,93	OK
Minimal reinf. ratio for secondary reinforcement (9.3.1.1 (2)) [%]	0,00	0,00	0,00	OK
Minimal clear distance of main reinforcement (8.2 (2)) [mm]	240	21	8,75	OK
Maximal axial distance of main reinforcement (9.3.1.1 (3)) [mm]	250	400	62,50	OK
Maximal axial distance of secondary reinforcement (9.3.1.1 (3)) [%]	0	450	0,00	OK

## 5. Literature

[1]	EN 1992-1-1: 2004 Eurocode 2 : design of concrete structures – Part 1: General rules and rules for building
[2]	ENV 1992-1-1: 1991 Eurocode 2 : design of concrete structures – Part 1: General rules and rules for building
[3]	Baumann, Th. : "Zur Frage der Netzbewehrung von Flächentragwerken". In : Der Bauingenieur 47 (1972), Berlin 1975