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High-Impact Instructional Practices in Mathematics

Ontario

Contents

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Components of Effective Math Instruction

Effective math instruction begins when educators have high expectations of students and believe that all students have the potential to learn and do math. It uses culturally relevant practices and differentiated learning experiences to meet individual students' learning needs. It focuses on the development of conceptual understanding and procedural fluency, skill development, communication, and problem-solving skills. And it involves educators choosing from and using a variety of high-impact instructional practices (Hattie, 2009; National Council of Teachers of Mathematics, 2014).

This resource is intended to build a common language and understanding about these high-impact practices in mathematics. It is designed to build awareness and expand repertoires to enhance teaching and learning. The resource comprises a series of fact sheets, described in more detail below, and concludes with a list of references and resources for further reading.

Before considering these specific practices, it is helpful to reflect upon the other factors that are important to math instruction – the importance of knowing the learner and understanding the curriculum, of providing a positive and safe learning environment, and of using various forms of assessment.

Knowing the Learner and the Curriculum

Effective instruction in all subjects requires that educators know their students, including their learning needs and strengths, their backgrounds and circumstances, and their social and personal identities. Teachers also need to be aware of their own "social location" – that is, who they are in terms of gender, race or ethnicity, socio-economic status, age, ability, religion, sexual orientation, and geographic location – and how this affects their ability to connect with their students. Educators' awareness of social location is important in teaching and learning because it acknowledges that all people do not think alike or experience reality in the same way – our perceptions are shaped by social factors. This awareness is essential to providing culturally responsive and relevant programs, and to enhancing students' overall sense of well-being and identity, and their ability to learn.

Through conversations and interactions with students, educators learn about their students' backgrounds, interests, strengths and assets, and needs – as well as about their mathematical understanding and thinking. They build an understanding of and appreciation for the unique perspective of students from linguistically diverse communities and of students with special education needs. Knowing where students are in their learning and what is needed to move their learning forward is the purpose of assessment and is essential for effective instruction.

With this information in mind, educators start planning with the curriculum and intentionally develop lessons and assessments together so that instruction and assessment are seamlessly integrated. They use purposeful, culturally relevant math tasks that build conceptual understanding and procedural knowledge, and they support students in making connections. Furthermore, effective educators understand how learning progresses through the grades, and how the curriculum expectations reflect this development.

Creating a Positive and Safe Learning Environment

Effective math instruction must be supported by an inclusive, positive, and safe learning environment, where students feel valued and engaged. To establish such an environment, educators inform students what is expected of them and how the classroom operates. When an assumption is made that students already know what is expected of them, ongoing issues with classroom management and engagement can occur. Developing norms and routines at the beginning of a period of instruction takes time, but it is time well spent that sets the stage for a productive year ahead. Moreover, when an educator decides to co-create classroom norms and routines with their students, the educator and students work together to build a positive learning environment.

In an effective environment for math instruction, students know what it "looks like, sounds like, and feels like" to:

- work with a partner or small group
- stick with a challenging problem
- represent and communicate their thinking
- use manipulatives
- work independently
- solve problems and reason logically
- listen actively
- give and receive feedback

The Role of Assessment

Assessment is the process of gathering information that accurately reflects how well a student is achieving the curriculum expectations in a subject. The primary purpose of assessment is to improve student learning. Assessment for the purpose of improving student learning is seen as both "assessment *for* learning" and "assessment *as* learning". Assessment *for* learning is the ongoing process of looking for evidence of each individual student's progress towards achieving the intended learning goals, and using this information to inform next steps. Assessment *as* learning refers to the student's participation in assessing their own progress and reflecting on their learning.

Observations, conversations, the practice of "noticing and naming the learning", portfolios, interviews, performance tasks, quizzes, and tests are just some of the assessment strategies and tools that math educators use to collect evidence of students' achievement of the provincial curriculum expectations. A balance of these strategies and tools can provide a clear picture of students' learning. Ultimately, effective assessment reveals where learners are, where they need to be, and how they will get there (Black and Wiliam, 2009). It informs when and how a particular instructional strategy can help consolidate or extend students' understanding. In other words, effective assessment lets educators strike the perfect balance between the challenge of a task and a student's readiness to perform it, sometimes referred to as cultivating "flow" – a state that optimizes engagement, productivity, and well-being (Liljedahl, 2018b).

When educators use a range of assessment practices to understand how their students are progressing, when they understand the curriculum and how learning develops, and when they create a positive classroom environment, they are forming a strong foundation for effective math instruction.

High-Impact Instructional Practices

The thoughtful use of high-impact instructional practices – including knowing when to use them and how they might be combined to best support the achievement of specific math goals – is an essential component of effective math instruction. This resource focuses on practices that researchers have consistently shown to have a high impact on teaching and learning mathematics (see meta-analysis by Hattie, Fisher, Frey, et al., 2017). Provided below is a series of fact sheets that describe the practices, what they look like in the classroom, and how they might be implemented:

- Learning Goals, Success Criteria, and Descriptive Feedback
- Direct Instruction
- Problem-Solving Tasks and Experiences
- Teaching about Problem Solving
- Tools and Representations
- Math Conversations

- Small-Group Instruction
- Deliberate Practice
- Flexible Groupings

While a lesson may prominently feature one of these high-impact practices, other practices will inevitably be involved. The practices are rarely used in isolation. Nor is there any single "best" instructional practice. Instead, educators choose the right practice, for the right time, in order to create an optimal learning experience for their students. They use their knowledge of their students, a deep understanding of the curriculum and of the mathematics that underpins the expectations, and a variety of assessment strategies to determine which particular high-impact instructional practice, or combination of practices, best supports their students.

These are significant decisions, with significant impact, that are made continually throughout a lesson. The hope is that this resource will spark conversations about these types of instructional decisions and the professional judgements that are so much a part of effective instruction. Teaching makes a difference, and the appropriate use of high-impact practices plays an important role in maximizing that difference.

Learning Goals, Success Criteria, and Descriptive Feedback

While some high-impact practices may be used on an as-needed basis, it is always important to provide students with learning goals and success criteria. This is an *essential* practice for effective instruction. Learning goals and success criteria outline the intention for the lesson and how this intention will be achieved. When educators and students have a clear and common understanding of what is being learned and what this learning looks like, all other instructional practices are stronger.

Although the learning goal does not have to be communicated to students at the *start* of every lesson, the educator has a clear understanding of the goal and is conscious of it throughout a lesson. By the end of a lesson, during consolidation, students will also be aware of it.

When the success criteria are clear and meaningful for students, they know what achieving the learning goal looks like and can monitor their progress towards the goal. When educators co-construct the criteria with students, this engages them with the subject matter – in this case, mathematics – and empowers them to take ownership of their learning. Success criteria often use "I" statements, which activate the brain and encourage learning through imitation and empathic listening. When students use "I" statements, they recognize that *they* are the force acting on their learning (Hattie, Fisher, Frey, et al., 2017).

It is also important that descriptive feedback be aligned with the success criteria. In this way, the student is provided with the precise information they need in order to reach the intended learning goal. When there are multiple opportunities for feedback and follow-up, all students gain skills in assessing their own learning as they reflect on the success criteria.

IN THE CLASSROOM

All students do better when they know what to do and what the intended learning is, whether this information is available on chart paper, on a digital platform, or in students' notebooks. Having clearly articulated learning goals, and an understanding of the steps necessary to achieve them, supports *all* learners. Students are then able to answer questions such as: "What does a 'complete response' look like?" "What does it mean to 'show your work'?" "What does it mean to 'justify your thinking'?"

When students are *beginning* to learn about a concept, success criteria should:

- define the terms or vocabulary to be used
- capture key mathematical concepts as they emerge in the classroom

As students *progress* with their learning, success criteria should:

- define the learning goal in student-friendly language
- capture key mathematical concepts as they emerge in the classroom
- demonstrate necessary conventions

When students are *deep* in the learning process, success criteria should:

- define what successful attainment of learning goals looks like
- make connections to other strands or content areas
- promote self-reflection

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Direct Instruction

Direct instruction is a concise, intentional form of instruction. It uses clearly communicated learning goals, introduces models and representations in context, and incorporates questioning and brief activities. It verbalizes thought processes, defines and uses math vocabulary, and makes key concepts and connections explicit. Direct instruction checks for understanding, summarizes the experience, and provides feedback. It can involve the whole class, small flexible groups, or individual students. *Effective* direct instruction is not a lecture. It is not didactic, educator-led talking from the front of the classroom (Hattie, 2009).

Direct instruction can vary in duration, depending on the grade or purpose of the instruction. It can take two minutes or twenty minutes, but it is always carefully planned to model, clarify, and extend mathematical thinking.

Effective direct instruction begins with a clear intention for the learning and identified success criteria. The students are engaged as the educator models, labels, questions, and checks for understanding. Direct instruction involves guided investigation, guided practice, feedback, and a consolidation that connects ideas, concepts, and skills from the lesson. It ends with an opportunity for students to practise, whether independently, with a partner, or in a small group. Once a concept or skill is learned, practice reinforces the concept or skill and supports the student in making connections.

IN THE CLASSROOM

Direct instruction might see students gathered on the carpet, seated at their desks, or standing at vertical non-permanent surfaces (e.g., whiteboards) as the educator models a specific strategy or use of a tool, asks questions, or monitors student understanding. The educator uses questioning, think-alouds, and different representations to move students toward the intended learning goal.

When students are *beginning* to learn about a concept, direct instruction should:

- activate prior knowledge and introduce new vocabulary
- highlight key mathematical ideas from previous student work
- connect different representations and strategies
- model how to use manipulatives or representations

As students *progress* with their learning, direct instruction should:

- reinforce procedures, or help students use more efficient procedures
- highlight or introduce mathematical conventions

When students are *deep* in the learning process, direct instruction should:

- highlight connections between tasks, strategies, representations, and concepts
- encourage metacognition or thinking about one's own thinking. When students are invited to think about and monitor their own thinking, they will develop a measure of "reasonableness" in their work that will enable them to evaluate it. Such metacognitive skills may not come naturally to all students, so there is a need to teach these skills.

Problem-Solving Tasks and Experiences

It is an effective practice to use problem-solving tasks and experiences to introduce concepts, build on prior knowledge, incorporate students' ideas, and consolidate learning. Problem-solving tasks and experiences can provide opportunities for students to reason, communicate, represent, and connect, as well as to justify their thinking. By inviting students to engage in these mathematical processes, the educator can determine students' current mathematical understanding, highlight key concepts, and lay the foundation for new math learning. The problems should be carefully selected and differentiated so that they are accessible yet challenging for all students – for example, parallel tasks using different numbers can enable students to work with the same problem at different levels.

When tasks have multiple entry points and allow for a variety of solution strategies, they are accessible to students at various stages of readiness, and give more students an opportunity to construct mathematical ideas. Engaging with a problem early in the learning process has proved more effective in improving understanding than "tell-then-practise" approaches (DeCaro and Rittle-Johnson, 2012; Loehr, Fyfe, and Rittle-Johnson, 2014).

During an effective consolidation of this experience, the educator facilitates a focused math discussion. In contrast to "show and tell", specific strategies and thinking are brought forward in support of the learning goal and development of the success criteria. When educators anticipate what might emerge in the classroom, they are better prepared to select student samples; annotate, sequence, and connect student work to highlight key mathematical concepts; and move the math learning forward.

IN THE CLASSROOM

A problem-solving experience can take many forms and is recognized by different names, such as a "three-part lesson", "three-act math", and a "thinking classroom". Essentially, it is an opportunity for students to activate prior knowledge, work collaboratively on a problem, and then discuss and share their strategies. This model of "activate, investigate, consolidate" supports collaboration and communication and provides an opportunity for students to construct a deeper understanding of math concepts. Research confirms that when students have the opportunity to construct some understanding *prior to* direct instruction, their learning is enhanced.

When students are *beginning* to learn about a concept, problem-solving experiences should:

- activate prior knowledge and be relevant to students' lived experiences
- offer multiple entry points and involve multiple solutions and/or solution strategies

As students *progress* with their learning, problem-solving experiences should:

- support student-generated procedures and invite students to choose an effective strategy
- provide an opportunity to represent thinking using concrete or pictorial models

When students are *deep* in the learning process, problem-solving experiences should:

- offer multiple entry points, involve multiple steps, and require justification of thinking
- invite students to compare and contrast tasks
- involve connections to other strands and content areas (e.g., cross-curricular applications of STEM) to support a transfer of learning

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Teaching about Problem Solving

Teaching students about the *process* of problem solving makes explicit the thinking that problem solving requires. It helps students to engage in "self talk", which they can use when facing a novel problem. It also helps students to understand the overall structure of a problem, and reinforces that problem solving requires perseverance and that a growth mindset is important.

Teaching about problem solving is often thought of as simply applying the four-step model: understand the problem, make a plan, carry out the plan, and reflect. But teaching about problem solving is more complex than this. For example, "understanding the problem" involves determining and analysing what information has been provided, determining what information is needed from another source (e.g., data or background knowledge), and determining what information the problem is asking us to find. It asks us to consider: "What do I know? What can I bring to this problem? What do I need to know?"

Understanding the problem is much more than underlining key words. In fact, using a "key word strategy" can even obscure an understanding of what is actually happening in the problem. Instead, teaching about problem solving involves teaching students how to represent the actions and quantities involved in the problem and identify what is known and what is unknown. Furthermore, problem solving involves looking for common *structures* that lie beneath the problem situation. To struggling students, each problem looks completely different. Educators need to support students as they generalize beyond "today's problem" to see connections *between* problems and recognize *types* of problems (Carpenter et al., 2015).

Teaching students about problem solving means talking to them about productive struggle, about making mistakes, and about adaptive reasoning. By valuing "struggle", educators can help students understand that difficulties, misconceptions, and errors are a natural part of learning. When educators value perseverance, they support their students in developing a growth mindset. Effective teaching about problem solving invites learners to reflect on their own thinking, to make unconscious processes explicit and available for them to use.

IN THE CLASSROOM

For all grades and learning abilities, teaching *about* problem solving involves talking about what is going on in our heads. Educators can model this by using think-alouds, by inviting students to share their thinking, by asking questions that encourage students to reflect on their own thinking, and by making the process explicit. Anchor charts, annotated student work, and digital platforms can be used to record the process and support students when solving new problems.

When students are *beginning* to learn about a concept, teaching about problem solving should:

- focus on helping students to understand the problem by first recognizing what information is provided and what it is they are being asked to do
- explore the underlying structure of problems
- discuss mistakes as an important part of learning

As students *progress* with their learning, teaching about problem solving should:

- focus on using effective representations to model the problem-solving situation
- highlight strategies to explain thinking and justify solutions
- highlight underlying structures or types of problems
- discuss perseverance as a necessary part of problem solving and learning

When students are *deep* in the learning process, teaching about problem solving should:

- involve comparing and contrasting problems to support students in recognizing the *structure* of each problem and in being able to generalize about problems and see beyond "today's" problem
- invite students to reflect on their learning, the strategies they used, and the "self talk" that helped them solve problems

Tools and Representations

The use of tools and representations supports a conceptual understanding of mathematics at all grade levels. Chosen carefully, tools and representations provide a way for students to think through problems and then communicate their thinking. Tools and representations explicitly and visually represent math ideas that are abstract. When paired with discussion, they help to demonstrate concepts and thinking. Visual representations provide an opportunity to talk about math, to examine mathematical relationships, and to make the problem-solving process visible. It should not be assumed, however, that tools and representations will enable students to draw correct conclusions automatically. Connections between the representations and the relevant mathematical ideas must be made explicitly, since the mathematical ideas are not in the representations themselves but rather in students' *thinking* about the mathematics. Used effectively, tools and representations not only make math concepts accessible to a wide range of learners but also give the educator insight into students' thinking. Because tools and representations draw on spatial reasoning and build "beyond language bridges", they can be useful when teaching linguistically diverse students or students with special education needs (Moschkovich, 2012). They help educators and students work together to build procedures from conceptual understanding.

When instruction is focused on helping students understand a particular math concept, the educator and students can use various different representations to support learning. Students who can represent mathematical ideas in a variety of ways demonstrate a deeper understanding of these concepts, as each representation provides a different perspective. Collectively, multiple perspectives allow for a richer, more complete understanding (Tripathi, 2008).

IN THE CLASSROOM

Using tools and representations in the classroom may involve the educator offering a real-world context for a mathematical problem. It may involve students drawing pictures or modelling their thinking concretely as they work with a partner. It may involve the educator using numbers, graphs, or symbols to represent a student's thinking. Using tools and representations effectively means that different representations are valued, connections are made between different representations, and abstract ideas are made accessible by being presented visually.

When students are *beginning* to learn about a concept, tools and representations should:

- connect to prior knowledge and students' lived experiences
- model situations concretely or pictorially
- model student thinking

As students *progress* with their learning, tools and representations should:

- be introduced to model situations in new ways
- be connected to other tools and representations
- include those that will be appropriate for future problems (e.g., those that will work with larger numbers or that can be transferred to other situations)

When students are *deep* in the learning process, tools and representations should:

- model situations concretely, pictorially, or abstractly, as is developmentally appropriate
- be compared and contrasted with other representations

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Math Conversations

Effective math classrooms provide multiple opportunities for students to engage in meaningful math talk. Conversations about math build understanding as students listen and respond to their classmates' expression of mathematical ideas. Students may share their ideas with a partner or within a small group, in the context of whole class discussions, or in the course of questioning and specific math-talk routines.

Math conversations can appear as short daily routines to support mental math and visualization strategies. They can include a single computational question or a sequence of calculations. Educators may ask students to place a number on a number line, to describe how they saw a configuration of dots, or to compare and contrast expressions, shapes, or graphs. The discussions that result from these routines provide opportunities for students to defend their thinking, to reason, and to prove. With norms and routines in place, students can share their thinking, and their peers can add to it or respectfully disagree with it.

Math conversations may also result from educators asking good questions that move students' thinking forward, provoke discussion, or probe into specific concepts, skills, or representations. Asking questions to highlight key concepts engages students, but it takes careful planning. Educators develop good questions by understanding key mathematical concepts and by "doing the math" in advance. They pose open questions – questions that cannot be answered with a simple "yes" or "no". Open questions allow for multiple responses, invite additional discussion from students, and move the math conversation from educator-student interactions to student-to-student math dialogues. Such discussion and dialogue may be a new skill for some students and may take time and practice.

IN THE CLASSROOM

Math conversations may take place at any time during a lesson, from giving math talk prompts at the start of a lesson to posing an open question or parallel task that consolidates learning at the end of a lesson. Math conversations can focus on number routines, such as number talks or analysing number strings. When analysing graphs, images, and patterns, asking students questions like "What do you notice? What do you wonder?" or "Which one doesn't belong?" can help further strengthen under standing of math concepts and spark deeper classroom discussion. Students can be encouraged to speak with a partner during a problem-solving task or to speak to the class during consolidation of learning. All of these different types of math conversations support students in consolidating their understanding of math.

As students are *beginning* to learn about a concept, effective conversations should:

- activate prior knowledge and connect the current task to previous learning ("How is this like something you have done before?")
- gather information about students' current level of understanding and ways of knowing ("How can you show your thinking?" "What math words can describe this?")

As students *progress* with their learning, effective conversations should:

- make math explicit ("How have you shown your thinking?")
- probe thinking and require explanations ("How could you explain your thinking to someone just learning this?" "How do you know?" "Why did you represent the problem this way?")
- reveal understanding and/or misconceptions ("How did you solve this problem?" "Where did you get stuck?")

When students are *deep* in the learning process, effective conversations should:

- support connections and transfer to other strands/content areas ("Where can you see this math at home? In other places?" "What other math connects to this?")
- require justifications and/or explanations ("Would this always be true? How do you know?")
- promote metacognition ("What was the most challenging thing about this task?" "What would you do differently if you solved a similar task again?")

Small-Group Instruction

Small-group instruction is a powerful instructional strategy for moving student learning forward. It allows for targeted, guided math instruction that meets the learning needs of specific students at specific times. By working with small and flexible groups, whether they are homogenous or heterogenous, the educator is able to personalize conversations and address key concepts that need to be clarified in order to prevent gaps from developing, to close gaps that already exist, or to extend thinking.

Small-group instruction includes models and representations, guided practice, and feedback. Educators select tasks to draw students' attention to specific mathematical concepts and then ask questions to highlight these concepts. Small-group instruction can focus on a mathematical concept or a process, such as problem solving, reasoning, proving, or representing thinking. To successfully manage a small group and the other students in the class, educators should ensure that small-group lessons are brief. This may require that learning goals be broken down or addressed over several mini-lessons.

IN THE CLASSROOM

While small-group instruction is widely recognized as a very effective practice, a question that educators often ask is: "What is the rest of the class doing while I am focusing on a small group of students?" Small-group instruction is so powerful at moving student thinking forward that it is worthwhile for educators to intentionally carve out time to include it in their daily plans. This means that the rest of the class will require clearly laid out learning experiences that are differentiated, flexible, and open, so that students can work on their own. The learning experiences may look different in different grades or at different stages of learning – for example, the rest of the class could be working at purposeful math stations that offer deliberate practice, playing a math game that utilizes a specific skill, collaboratively solving a problem with a partner, or working independently. Regardless of the particular tasks the rest of the class is working on, established classroom norms and routines are necessary to ensure that small-group instructional time is not interrupted.

When students are *beginning* to learn about a concept, small-group instruction should:

- revisit math concepts that support the new learning
- activate students' prior knowledge by making connections to their lived experiences

As students *progress* with their learning, small-group instruction should:

- reinforce understanding through the use of representations
- involve comparing and contrasting problems and examining problem structures

When students are *deep* in the learning process, small-group instruction should:

- extend students' thinking and encourage the transfer of skills to other math concepts and strands
- support metacognition by inviting students to think about their learning

Deliberate Practice

Practice is a necessary component of an effective math program. Practice is best when it is deliberate, purposeful, and spaced, and it can take many forms – math games, math stations, and paper-and-pencil tasks – any of which can be done independently or with a partner. Regardless of the form of practice, ongoing feedback is crucial, so that students know that they are practising correctly and that they have practised sufficiently. This ensures that practice is as effective as possible.

While skill development is part of effective practice, it is not the only thing that students need to work on. Students need to practise representing their thinking, problem solving, and communicating their thinking. Such practice strengthens the connection between skills, concepts, and strategies. Students also need to practise metacognition, or reflecting on their learning. With metacognition, learning becomes self-directed. When a student thinks, "I think I get it now, but I need a little more practice to feel comfortable doing it on my own", the student has taken ownership of their learning.

It is important that practice *follows* understanding. Without understanding, students may unwittingly practise misconceptions. Additionally, if the focus is on the practice of procedures *early* in the learning, then students will focus on efficient procedures at the *expense* of understanding (Hattie, Fisher, Frey, et al., 2017). Practice of procedures should occur after students have a good understanding of concepts.

IN THE CLASSROOM

Practice can take many forms. A math game can provide an opportunity for students to practise a skill. A math station with two-dimensional shapes, paper, and some tape can be an opportunity for students to practise constructing solids. Regular opportunities for students to talk with a partner can allow them to practise representing and communicating their thinking. A review of problems from previous units can be an opportunity for spaced or distributed practice of concepts and skills. When it is deliberate, purposeful, spaced, and paired with feedback, practice has a positive impact on learning.

When students are *beginning* to learn about a concept, deliberate practice should:

- revisit prior knowledge or supporting concepts
- affirm students' progress and positively support growth in areas of challenge

As students *progress* with their learning, deliberate practice should:

- strengthen skills, concepts, and mathematical processes

When students are *deep* in the learning process, deliberate practice should:

- make connections between skills, concepts, and processes

Flexible Groupings

Flexible groupings promote collaboration and provide opportunities for students to engage in rich mathematical conversations, learn from one another, and move their mathematical thinking forward. Creating flexible groupings in a math class enables students to work independently of the teacher, but with the support of their peers. It is the intentional combination of large-group, small-group, partner, and independent work experiences that can foster a rich mathematical learning environment. *Flexible* collaborative groups work best because students need to be grouped differently for different reasons.

Whole-class instruction provides an opportunity for shared experiences. It enables students to hear a broad range of strategies and perspectives and provides a rich and diverse opportunity for math discourse.

Small-group and partner experiences offer students a low-risk environment to explore mathematical ideas. Small groups can reflect random, heterogeneous, or homogeneous pairings. Random groupings give students the opportunity to work with new people, and expose them to a variety of strategies and ways of thinking. Mixed-ability groupings can support all students, including linguistically diverse students and students with special education needs. Such groupings enable the educator to move more freely through the classroom to observe, prompt, and monitor student thinking. They also expose students to a variety of thinking strategies and vocabulary. In like-ability groups, students have similar understandings of the math, which enables the educator to offer differentiated tasks, targeted intervention, and enrichment tasks.

Partner or small-group experiences can be more effective than larger groupings at engaging all learners. Whatever the grouping, accountability measures need to be in place. Students need to recognize that they will be asked to describe and defend their partner's thinking. A think-pair-share strategy, different coloured markers, and assigned roles are some ways to build accountability into partner or small-group work.

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A variety of groupings may be used over the course of a lesson. Random groups may be created at the start of the lesson, as students engage in a task that activates prior knowledge. This may be followed by homogeneous pairings as students work on a problem-solving task and the teacher works with a small group. The lesson may conclude with heterogeneous pairs playing a game, followed by independent practice.

When students are *beginning* to learn about a concept, flexible groupings should:

- expose students to a variety of strategies and offer peer support as the educator moves through the classroom to observe and assess next steps

As students *progress* with their learning, flexible groupings should:

- allow for the use of parallel tasks that meet students where they are
- expose students to specific representations, concepts, and skills to advance learning

When students are *deep* in the learning process, flexible groupings should:

- encourage reflection and connections between mathematical concepts
- consolidate and confirm learning

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