THE ONTARIO CURRICULUM

GRADES 1–8

MATHEMATICS

Curriculum Context

2020



Ontario

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Une publication équivalente est disponible en français sous le titre suivant : *Le curriculum de l'Ontario, de la 1re à la 8e année – Mise en contexte du programme-cadre de mathématiques* (2020).

This publication is available on the Ministry of Education's website, at ontario.ca/curriculum.

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Preface

This document is a PDF version of the curriculum context for the learning expectations published on the Ministry of Education's Curriculum and Resources site, at ontario.ca/curriculum. Together, the context, expectations, and teacher supports comprise *The Ontario Curriculum, Grades 1–8: Mathematics, 2020*, a revised curriculum policy replacing *The Ontario Curriculum, Grades 1–8: Mathematics, 2005*. Effective September 2020, all mathematics programs for Grades 1 to 8 will be based on the expectations outlined on the site.

The Ontario Curriculum, Grades 1–8: Mathematics, 2020 focuses on fundamental mathematics concepts and skills, as well as on making connections between related math concepts, between mathematics and other disciplines, and between mathematics and everyday life. It also supports new learning about mathematical modelling, coding, and financial literacy, and integrates mathematics learning with learning in other STEM (science, technology, engineering, and mathematics) subjects. As well, this curriculum is designed to help students build confidence in approaching mathematics and acquire a positive attitude towards mathematics, cope with stress and anxiety, persevere and learn from their mistakes, work collaboratively with others towards a shared goal, value deep thinking and making connections, and become capable and confident math learners.

Vision and Goals

Recent research and practice have provided a clearer understanding of how students learn mathematical concepts and skills. In addition, technology has changed how we access information and how students interact with mathematics. All students bring to school their mathematical experiences learned in various contexts. Schools should take advantage of these various experiences so that mathematics classrooms become places of diverse and inclusive learning that value multiple ways of knowing and doing. These places will allow all students to become flexible and adaptive learners in an ever-changing world. The vision of the mathematics curriculum is to help all students develop a positive identity as a mathematics learner and see themselves as mathematically skilled, to support them as they use mathematics to make sense of the world, and to enable them to make critical decisions based on mathematically sound principles. This vision is attained in a mathematics classroom filled with enthusiasm and excitement – a classroom where all students receive the highest-quality mathematics instruction and learning opportunities, interact as confident mathematics learners, and are thereby enabled to reach their full potential.

Success in mathematics has often been viewed as an important indicator of career success. The goal of the Ontario mathematics curriculum is to provide all students with the foundational skills required to:

- understand the importance of and appreciate the beauty of mathematics;
- recognize and appreciate multiple mathematical perspectives;
- make informed decisions and contribute fully to their own lives and to today's competitive global community;
- adapt to changes and synthesize new ideas;
- work both independently and collaboratively to creatively approach challenges;
- communicate effectively;
- think critically and creatively and see connections to other disciplines beyond mathematics, such as other STEM disciplines.

In order to develop a strong understanding of mathematics, all students must feel that they are connected to the curriculum. They must see themselves in what is taught, in why it is taught, and in how it is taught. They must also see how their learning applies to their own context and to the world. The needs of learners are diverse, and all learners have the capacity to develop the knowledge, concepts, skills, and perspectives they need to become informed, productive, and responsible citizens in their own communities and in the world.

How mathematics is contextualized, positioned, promoted, discussed, taught, learned, evaluated, and applied affects all students. Mathematics must be appreciated for its innate beauty, as well as for its role in making sense of the world. Having a solid foundation in mathematics and a deep appreciation for and excitement about mathematics will help ensure that all students are confident and capable as they step into the future.

The Importance and Beauty of Mathematics

Mathematics is integral to every aspect of daily life – social, economic, cultural, and environmental. It is part of the story of human history. People around the world have used and continue to use mathematical knowledge, skills, and attitudes to make sense of the world around them and develop new mathematical thinking and appreciation for mathematics. The relationships between cultures and mathematics are conceptualized and practiced in many different ways across many different contexts. From counting systems, measurement, and calculation to arithmetic, geometry, and spatial sense, mathematics has been evident in the daily lives of people across history.

Today, mathematics continues to be found all around us. For example, mathematics can be found in medicine, sports performance analysis, navigation systems, electronic music production, computer gaming, quantum physics, fashion design, and so much more. Mathematics skills are necessary when we buy goods and services online, complete our taxes, create art, and play sports. Mathematics also exists in nature, storytelling, puzzles, and games. Proficiency with mathematical ideas is needed for many careers, including but not limited to engineering, health care and medicine, computer science, finance, landscape design, architecture, agriculture, the arts, the culinary arts, and many skilled trades. In fact, in every field of pursuit, the analytical, problem-solving, critical-thinking, and creative-thinking skills that students develop through the study of mathematics are evident. In the modern age of evolving technologies, artificial intelligence, and access to vast sources of information and big data, knowing how to navigate, interpret, analyse, reason, evaluate, and problem solve is foundational to everyday life.

While mathematics can be understood as a way of studying and understanding structure, order, and relationships, the aesthetics of mathematics have also motivated the development of new mathematical thinking. The power of mathematics is evident in the connections between seemingly abstract mathematical ideas. The applications of mathematics have often yielded fascinating representations and results. The beauty in mathematics can be found in the process of deriving elegant and succinct approaches to resolving problems. Other times, messy problems and seeming chaos may culminate in beautiful, sometimes surprising, results that are both simple and generalizable. Most important, the beauty of mathematics is experienced when exciting breakthroughs in problem solving are made

and an air of relief and awe is enjoyed. The two aspects of mathematics, aesthetics and application, are deeply interconnected.

The Ontario mathematics curriculum strives to equip all students with the knowledge, skills, and habits of mind that are essential to understanding and enjoying the importance and beauty of mathematics.

Learning in the mathematics curriculum begins with a focus on the fundamental concepts and foundational skills. This leads to an understanding of mathematical structures, operations, processes, and language that provides students with the means necessary for reasoning, justifying conclusions, and expressing and communicating mathematical ideas clearly. Through relevant and meaningful learning opportunities and through the strategic use of technology, all students are supported as they learn and apply mathematical concepts and skills within and across strands and other subject areas.

The Ontario mathematics curriculum helps establish an inclusive mathematical learning community where all students are invited to experience the living practice of mathematics, to work through challenges, and to find success and beauty in problem solving. As students engage with the curriculum, they may incorporate their prior experience and existing mathematical understanding, and then integrate the new ideas they learn into their daily lives. As all students see themselves reflected in what is taught and how it is taught, they begin to view themselves as competent and confident mathematics learners. As a result, they develop improved mathematical knowledge, concepts, and skills as well as an improved sense of mathematical agency and identity. This encouragement of mathematical confidence subsequently opens doors for all students to explore the importance and beauty of mathematics while they make connections to other subjects, explore the world, and later pursue further studies.

Principles Underlying the Ontario Mathematics Curriculum

The Ontario mathematics curriculum for Grades 1 to 8 is founded on the following principles:

- **A mathematics curriculum is most effective when it values and celebrates the diversity that exists among students.**

  The Ontario mathematics curriculum is based on the belief that all students can and deserve to be successful in mathematics. It recognizes that not all students necessarily learn mathematics in the same way, use the same resources, and/or learn within the same time frames. Setting high expectations and building a safe and inclusive community of learners requires the use of a variety of differentiated instructional and assessment strategies and approaches that create an optimal and equitable environment for mathematics learning.

- **A robust mathematics curriculum is essential for ensuring that all students reach their full potential.**

  The Ontario mathematics curriculum challenges all students by including learning expectations that capitalize on students' prior knowledge; involve higher-order thinking skills; and require students to make connections between their lived experiences, mathematical concepts, other subject areas, and situations outside of school. This learning enables all students to gain a powerful knowledge of the usefulness of the discipline and an appreciation of the importance of mathematics.

- **A mathematics curriculum provides all students with the foundational mathematics concepts and skills they require to become capable and confident mathematics learners.**

  The Ontario mathematics curriculum provides a balanced approach to the teaching and learning of mathematics. It is based on the belief that all students learn mathematics most effectively when they develop a solid understanding of the fundamental concepts and skills in mathematics and are given opportunities to apply these concepts and skills as they solve increasingly complex tasks and investigate mathematical ideas, applications, and situations in everyday contexts. As students begin to see the relevance of mathematics and to see themselves as capable mathematics learners, they begin to develop a positive identity as a mathematics learner.

- **A progressive mathematics curriculum includes the strategic integration of technology to support and enhance the learning and doing of mathematics.**

  The Ontario mathematics curriculum strategically integrates the use of appropriate technologies to help all students develop mathematical knowledge, concepts, and skills, while recognizing the continuing importance of students' mastering the fundamentals of mathematics. For some students, assistive technology also provides an essential means of accessing the mathematics curriculum and demonstrating their learning. Students develop the ability to select appropriate tools and strategies to perform particular tasks, to investigate ideas, and to solve problems. The curriculum sets out a framework for learning important skills, such as problem solving, coding, and modelling, as well as opportunities to develop critical data literacy skills, information literacy skills, and financial literacy skills.

- **A mathematics curriculum acknowledges that the learning of mathematics is a dynamic, gradual, and continuous process, with each stage building on the last.**

  The Ontario mathematics curriculum is dynamic, continuous, and coherent and is designed to help all students develop an understanding of the universal coherence and nature of mathematics. Students see how concepts develop and how they are interconnected. Teachers observe and listen to all students and then responsively shape instruction in ways that will foster and deepen student understanding of important mathematics. The fundamental concepts, skills, and processes are introduced in the primary grades and solidified and extended throughout the junior and intermediate grades. The program is continuous, as well, from the elementary to the secondary level. Teachers connect mathematics to students' everyday experiences, which helps all students develop a deeper understanding of the relevance of mathematics to the world beyond the classroom. Students also come to understand that learning in mathematics never ends.

- **A mathematics curriculum is integrated with the world beyond the classroom.**

  The Ontario mathematics curriculum provides opportunities for all students to investigate and experience mathematical situations they might find outside of the classroom and develop an appreciation for the beauty and wide-reaching nature and importance of mathematics. The overall program integrates and balances concept development and skill development, including social-emotional learning skills, as well as the use of mathematical processes and real-life applications.

- **A mathematics curriculum motivates students to learn and to become lifelong learners.**

  The Ontario mathematics curriculum is brought to life in the classroom, where students develop mathematical understanding and are given opportunities to relate their

knowledge, concepts, and skills to wider contexts. Making connections to the world around them stimulates their interest and motivates them to become lifelong learners with positive attitudes towards mathematics. Teachers bring the mathematics curriculum to life using their knowledge of:

- the mathematics curriculum;
- the backgrounds and identities of all students, including their past and ongoing experiences with mathematics, learning strengths, and needs;
- mathematical concepts and skills, and how they are connected across the strands and with other disciplines;
- instructional approaches and assessment strategies best suited to meet the learning needs of all students;
- resources designed to support and to enhance the achievement of and engagement with the curriculum expectations, while fostering an appreciation for and joy in math learning.

Roles and Responsibilities in Mathematics Education

Students

It is essential that all students take responsibility for their own learning as they progress through elementary and secondary school. Mastering the skills and concepts connected with learning in the mathematics curriculum requires a commitment to learning that includes:

- continual and consistent personal reflection and goal setting
- a belief that they are capable of succeeding in mathematics
- developing skills in persevering when taking on new challenges
- connecting prior experiences, knowledge, skills, and habits of mind to new learning
- a willingness to work both collaboratively and independently
- dedication to ongoing practice
- an ability to receive and respond to meaningful feedback

Through ongoing practice and reflection, all students can develop a positive and healthy mathematical identity whereby they value and appreciate mathematics as a discipline, see themselves as mathematics learners, and understand what successful math learning looks like.

Students' attitudes towards mathematics education can have a significant impact on their engagement with math learning and their subsequent learning and achievement of the expectations. Students who are engaged in their learning and who have opportunities to solve interesting, relevant, and meaningful problems within a supportive, safe, and inclusive learning environment are more likely to adopt practices and behaviours that support mathematical thinking. More importantly, they are more likely to enjoy mathematics and to pursue their desire to learn math beyond the classroom setting.

With teacher support and encouragement, students learn that they can apply the skills they acquire in mathematics to other contexts and subjects. For example, they can apply the problem-solving skills they use in mathematics to their study of the science and social studies curricula. They can also make connections between their learning and life beyond the classroom. For example, when reading or watching the news, they can look for

applications of mathematical modelling and how it can be used to answer important questions related to global health and the environment or to help solve critical social issues that are relevant to their lives and experiences.

Parents

Parents[1] are their children's first role models. It is important for schools and parents to work together to ensure that home and school provide a mutually supportive framework for young people's mathematics education. Research assures us of the positive results of parent engagement on student success – and parent-child communication about mathematics, including parents' fostering of positive attitudes towards mathematics, is one of the many important ways parents may be involved.

Parents can support their children's mathematics success by showing an interest in what their children are learning and by discovering with their children how what is being learned in class can be applied to everyday contexts. Math is everywhere, and parents can help their children make connections between what they are learning at school and everyday experiences at home and in the community – such as cooking at home, shopping at a store, and managing household finances. Parents can include their children when cooking at home by asking them to measure ingredients and to double or halve a recipe. They can include their children when making decisions at the grocery store by asking them to figure out what is the better deal and to estimate the total cost of items in their cart before proceeding to checkout. They can include their children in other ways – for example, when enjoying math-based puzzles and games – and they can create opportunities for mental math estimations and calculations and for making predications. Parents can support their children's learning by encouraging them to complete their mathematics tasks, to practise new skills and concepts, to apply new mathematics learning to experiences at home, and to connect mathematical experiences at home to learning at school.

More importantly, parents are an integral part of their children's interactions and experiences with mathematics. Having a positive attitude towards mathematics and developing self-efficacy are important elements of students' achievement of the expectations and of all future mathematics learning. By demonstrating a positive attitude towards mathematics, and by speaking positively and often about mathematics, parents can show their children that mathematics is enjoyable, worthwhile, and valuable. Parents can encourage their children to cultivate perseverance when solving problems, to acknowledge any difficulties, to believe that they can succeed in math, and to build their own self-confidence and sense of identity as mathematics learners.

1. The word *parent(s)* is used in this curriculum to refer to parent(s) and guardian(s). It may also be taken to include caregivers or close family members who are responsible for raising the child.

Schools offer a variety of opportunities for parents to learn more about how to support their children: for example, events related to mathematics may be held at the school (e.g., family math nights); teachers may provide newsletters or communicate with parents through apps or social media; and school or board websites may provide helpful tips about how parents can engage in their child's mathematics learning outside of school and may even provide links where they can learn more or enjoy math activities together.

If parents need more information about what their children are learning, and how to support their children's success in mathematics, teachers are available to answer questions and provide information and resources.

Teachers

Teachers are critical to the success of students in mathematics. Teachers are responsible for ensuring that all students receive the highest quality of mathematics education. This requires them to have high expectations of all students and to view all students as capable math learners. Teachers bring enthusiasm and skill in providing varied and equitable instructional and assessment approaches to the classroom, addressing individual students' identities, profiles, strengths and needs, and ensuring equitable, accessible, and engaging learning opportunities for every student. The attitude with which teachers themselves approach mathematics is critical, as teachers are important role models for students.

Teachers place students at the centre of their mathematics planning, teaching, and assessment practices, and understand how the learning experiences they provide will develop a love of mathematics and foster a positive "I can do math" attitude in all students. Teachers have a thorough understanding of the mathematics content they teach, which enables them to provide relevant and responsive opportunities through which all students can develop their understanding of mathematical knowledge, concepts, and skills. Teachers understand the learning continuum along which students develop their mathematical thinking and can thus support all students' movement along this continuum. Teachers support students in developing their ability to solve problems, reason mathematically, and connect the mathematics they are learning to the real world around them. Teachers provide ongoing meaningful feedback to all students about their mathematics achievement, which helps to build confidence. They recognize the importance of emphasizing the usefulness of mathematics in students' lives, and of integrating mathematics with other areas of the curriculum – such as making connections with science, engineering, and technology to answer scientific questions or solve problems. They recognize the importance of helping students learn about careers involving mathematics, and of supporting the development of a positive attitude towards mathematics and student mathematical agency.

As part of effective teaching practice, teachers communicate with parents, using multiple ways and by both formal and informal means to meet the diverse needs of families, and to

better understand students' mathematical experiences outside of the school. In addition, teachers discuss with parents what their children are learning in mathematics at school. Communication enables parents to work in partnership with the school, leading to stronger connections between the home and school to support student learning and achievement in mathematics.

Principals

Principals model the importance of lifelong learning, and of how mathematics plays a vital role in the future success of students. Principals provide support for the successful implementation of the mathematics curriculum by emphasizing the importance of mathematics, by promoting the idea that all students are capable of becoming confident mathematics learners, and by encouraging a positive and proactive attitude towards mathematics and student agency in mathematics.

The principal works in partnership with teachers and parents to ensure that all students have access to the best possible educational experience. To support student learning, principals monitor the implementation of the Ontario mathematics curriculum. Principals ensure that English language learners are being provided the accommodations and/or modifications they require for success in the mathematics program. Principals are also responsible for ensuring that every student who has an Individual Education Plan (IEP) is receiving the modifications and/or accommodations described in their plan – in other words, for ensuring that the IEP is properly developed, implemented, and monitored.

Ensuring that teachers have the agency, support, confidence, resources, and tools they need to deliver a high-quality program is essential. Principals collaborate with teachers and school and system leaders to develop professional learning opportunities that deepen teachers' knowledge of the curriculum, mathematical content, and pedagogy, and enhance their self-efficacy in teaching mathematics. Additional professional learning and support to increase teachers' knowledge, awareness, and comfort level in teaching mathematics may be provided by principals where necessary.

Community Partners

Community partners are an important resource for a school's mathematics education program. Relationships with local businesses, volunteer groups, and community organizations, such as those for newcomer families, can provide opportunities for authentic perspectives and real-world application of mathematics. Nurturing partnerships with other schools can facilitate the sharing of resources, strategies, and facilities; the development of professional learning opportunities for staff; and the hosting of special events such as family math nights or a mathematics community walk.

Communities provide social contexts for learning. Students bring knowledge and experiences from their homes and communities that can be powerful assets in creating productive environments for learning. By involving others in the community, teachers and principals can position mathematics learning as collaborative and experiential. Membership in a community also helps students develop a sense of identity and belonging and build their identity as mathematics learners.

The Program in Mathematics

Curriculum Expectations

The Ontario Curriculum, Grades 1–8: Mathematics, 2020 identifies the expectations for each grade and describes the knowledge, concepts, and skills that students are expected to acquire, demonstrate, and apply in their class work and activities, on tests in demonstrations, and in various other activities on which their achievement is assessed and evaluated.

Mandatory learning is described in the overall and specific expectations of the curriculum.

Two sets of expectations – overall expectations and specific expectations – are listed for each *strand*, or broad area of the curriculum, in mathematics for Grades 1 to 8. The strands include Strand A and five strands, lettered B, C, D, E, and F. *Taken together, the overall and specific expectations represent the mandated curriculum*.

The *overall expectations* describe in general terms the knowledge, concepts, and skills that students are expected to demonstrate by the end of each grade. The *specific expectations* describe the expected knowledge, concepts, and skills in greater detail. The specific expectations are grouped under numbered subheadings, each of which indicates the strand and the overall expectation to which the group of specific expectations corresponds (e.g., "B2" indicates that the group relates to overall expectation 2 in strand B). This organization is not meant to imply that the expectations in any one group are achieved independently of the expectations in the other groups, nor is it intended to imply that learning the expectations happens in a linear, sequential way. The numbered headings are used merely as an organizational structure to help teachers focus on particular aspects of knowledge, concepts, and skills as they develop various lessons and learning activities for students. In the mathematics curriculum, strands B to F use additional subheadings within each group of expectations to identify the topics addressed in the strand.

In the mathematics curriculum, the overall expectations outline the fundamental knowledge, concepts, and skills that are required for engaging in appropriate mathematical situations in and out of the classroom at any grade or stage of development. For this reason, the overall expectations generally remain the same from Grades 1 to 8. The curriculum focuses on connecting, developing, reinforcing, and refining the knowledge, concepts, and skills

that students acquire as they work towards meeting the overall expectations in the elementary school program. This approach reflects and accommodates the progressive nature of development of knowledge, concepts, and skills in mathematics learning.

The *specific expectations* reflect this progression in knowledge and skill development through changes in the wordings of the expectations and through the introduction of new expectations, where appropriate. The progression is captured by the increasing complexity of the pedagogical supports (see below) associated with most expectations and by the increasing specificity of mathematical relationships, the diversity of contexts in which the learning is applied, and the variety of opportunities presented for applying it. It should be noted that *all* the skills specified in the early grades continue to be developed and refined as students move through the grades, whether or not each of those skills continues to be explicitly required in an expectation.

There is an exception in Strand C: Algebra, where the overall expectation on mathematical modelling has no accompanying specific expectations. This is because mathematical modelling is an integrated process that is applied in various contexts, allowing students to extend and apply what they have learned in other strands. Students' demonstration of the process of mathematical modelling, as they apply knowledge, concepts, and skills learned in other strands, is assessed and evaluated.

In addition to the expectations outlined within the other five strands, Strand A focuses on the development and application of social-emotional learning (SEL) skills while using mathematical processes. These skills support students' understanding of mathematical knowledge, concepts, and skills and foster their overall well-being and ability to learn while helping them build resilience and thrive as mathematics learners. As they develop SEL skills, students demonstrate a greater ability to understand and apply the mathematical processes, which are critical to supporting learning in mathematics. In all grades of the mathematics program, the learning related to this strand takes place in the context of learning related to the other five strands and is assessed and evaluated within these contexts.

Examples, Key Concepts, and Sample Tasks

Specific expectations are accompanied by examples, key concepts, and/or sample tasks. These elements or "pedagogical supports"[2] are intended to promote understanding of the intent of the specific expectations, and are offered as illustrations for teachers. *The pedagogical supports do not set out requirements for student learning; they are optional, not mandatory*.

2. The pedagogical supports will be made available as part of the full curriculum release this summer.

The examples are meant to illustrate the intent of the expectation, illustrating the kind of knowledge or skill, the specific area of learning, the depth of learning, and/or the level of complexity that the expectation entails. The key concepts identify the central principles and mathematical ideas that underpin the learning in that specific expectation. The sample tasks have been developed to model appropriate practice for the grade. They provide possible learning activities for teachers to use with students and illustrate connections between the mathematical knowledge, concepts, and skills. Teachers can choose to draw on the sample tasks that are appropriate for their classrooms, or they may develop their own approaches that reflect a similar level of complexity. Whatever the specific ways in which the requirements outlined in the expectations are implemented in the classroom, they must, wherever possible, be inclusive and reflect the diversity of the student population and the population of the province. Teachers will notice that some of the sample tasks not only address the requirements of the expectation they are associated with but also incorporate mathematical concepts or skills described in expectations in other strands in the same grade.

The Mathematical Processes

Students learn and apply the mathematical processes as they work to achieve the expectations outlined in the curriculum. All students are actively engaged in applying these processes throughout the program. They apply these processes, together with social-emotional learning (SEL) skills, across the curriculum to support learning in mathematics. See the section "Strand A: Social-Emotional Learning (SEL) Skills in Mathematics and the Mathematical Processes" for more information.

The mathematical processes that support effective learning in mathematics are as follows:

- problem solving
- reasoning and proving
- reflecting
- connecting
- communicating
- representing
- selecting tools and strategies

The mathematical processes can be seen as the processes through which all students acquire and apply mathematical knowledge, concepts, and skills. These processes are interconnected. Problem solving and communicating have strong links to all the other processes. A problem-solving approach encourages students to reason their way to a solution or a new understanding. As students engage in reasoning, teachers further encourage them to pose questions, make conjectures, and justify solutions, orally and in writing. The communication and reflection that occur before, during, and after the process of problem solving help students not only to articulate and refine their thinking but also to see the problem they are solving from different perspectives. This opens the door to recognizing the range of strategies that can be used to arrive at a solution. By seeing how others solve a problem, students can begin to reflect on their own thinking (a process known as "metacognition") and the thinking of others, as well as their own language use (a process known as "metalinguistic awareness"), and to consciously adjust their own strategies in order to make their solutions as efficient and accurate as possible.

The mathematical processes cannot be separated from the knowledge, concepts, and skills that students acquire throughout the year. All students problem solve, communicate,

reason, reflect, and so on, as they develop the knowledge, the understanding of mathematical concepts, and the skills required in all the strands in every grade.

Problem Solving

Problem solving is central to doing mathematics. By learning to solve problems and by learning *through* problem solving, students are given, and create, numerous opportunities to connect mathematical ideas and to develop conceptual understanding. Problem solving forms the basis of effective mathematics programs that place all students' experiences and queries at the centre. Thus, problem solving should be the mainstay of mathematical instruction. It is considered an essential process through which all students are able to achieve the expectations in mathematics and is an integral part of the Ontario mathematics curriculum.

Problem solving:

- increases opportunities for the use of critical thinking skills (e.g., selecting appropriate tools and strategies, estimating, evaluating, classifying, assuming, recognizing relationships, conjecturing, posing questions, offering opinions with reasons, making judgements) to develop mathematical reasoning;
- helps all students develop a positive math identity;
- allows all students to use the rich prior mathematical knowledge they bring to school;
- helps all students make connections among mathematical knowledge, concepts, and skills, and between the classroom and situations outside the classroom;
- promotes the collaborative sharing of ideas and strategies and promotes talking about mathematics;
- facilitates the use of creative-thinking skills when developing solutions and approaches;
- helps students find enjoyment in mathematics and become more confident in their ability to do mathematics.

Most importantly, when problem solving is in a mathematical context relevant to students' experiences and derived from their own problem posing, it furthers their understanding of mathematics and develops their math agency.

Problem-Solving Strategies. Problem-solving strategies are methods that can be used to solve problems of various types. Common problem-solving strategies include the following: simulating; making a model, picture, or diagram; looking for a pattern; guessing and checking; making an organized list; making a table or chart; solving a simpler version of the problem (e.g., with smaller numbers); working backwards; and using logical reasoning. Teachers can support all students as they develop their use of these strategies by engaging with solving various kinds of problems – instructional problems, routine problems, and non-routine problems. As students develop this repertoire over time,

they become more confident in posing their own questions, more mature in their problem-solving skills, and more flexible in using appropriate strategies when faced with new problem-solving situations.

Reasoning and Proving

Reasoning and proving are a mainstay of mathematics and involves students using their understanding of mathematical knowledge, concepts, and skills to justify their thinking. Proportional reasoning, algebraic reasoning, spatial reasoning, statistical reasoning, and probabilistic reasoning are all forms of mathematical reasoning. Students also use their understanding of numbers and operations, geometric properties, and measurement relationships to reason through solutions to problems. Teachers can provide all students with learning opportunities where they must form mathematical conjectures and then test or prove them to see if they hold true. Initially, students may rely on the viewpoints of others to justify a choice or an approach to a solution. As they develop their own reasoning skills, they will begin to justify or prove their solutions by providing evidence.

Reflecting

Students reflect when they are working through a problem to monitor their thought process, to identify what is working and what is not working, and to consider whether their approach is appropriate or whether there may be a better approach. Students also reflect after they have solved a problem by considering the reasonableness of their answer and whether adjustments need to be made. Teachers can support all students as they develop their reflecting and metacognitive skills by asking questions that have them examine their thought processes, as well as questions that have them think about other students' thought processes. Students can also reflect on how their new knowledge can be applied to past and future problems in mathematics.

Connecting

Experiences that allow all students to make connections – to see, for example, how knowledge, concepts, and skills from one strand of mathematics are related to those from another – will help them to grasp general mathematical principles. Through making connections, students learn that mathematics is more than a series of isolated skills and concepts and that they can use their learning in one area of mathematics to understand another. Seeing the relationships among procedures and concepts also helps develop mathematical understanding. The more connections students make, the deeper their understanding, and understanding, in turn, helps them to develop their sense of identity. In addition, making connections between the mathematics they learn at school and its

applications in their everyday lives not only helps students understand mathematics but also allows them to understand how useful and relevant it is in the world beyond the classroom. These kinds of connections will also contribute to building students' mathematical identities.

Communicating

Communication is an essential process in learning mathematics. Students communicate for various purposes and for different audiences, such as the teacher, a peer, a group of students, the whole class, a community member, or their family. They may use oral, visual, written, or gestural communication. Communication also involves active and respectful listening. Teachers provide differentiated opportunities for all students to acquire the language of mathematics, developing their communication skills, which include expressing, understanding, and using appropriate mathematical terminology, symbols, conventions, and models.

For example, teachers can ask students to:

- share and clarify their ideas, understandings, and solutions;
- create and defend mathematical arguments;
- provide meaningful descriptive feedback to peers; and
- pose and ask relevant questions.

Effective classroom communication requires a supportive, safe, and respectful environment in which all members of the class feel comfortable and valued when they speak and when they question, react to, and elaborate on the statements of their peers and the teacher.

Representing

Students represent mathematical ideas and relationships and model situations using tools, pictures, diagrams, graphs, tables, numbers, words, and symbols. Teachers recognize and value the varied representations students begin learning with, as each student may have different prior access to and experiences with mathematics. While encouraging student engagement and affirming the validity of their representations, teachers help students reflect on the appropriateness of their representations and refine them. Teachers support students as they make connections among various representations that are relevant to both the student and the audience they are communicating with, so that all students can develop a deeper understanding of mathematical concepts and relationships. All students are supported as they use the different representations appropriately and as needed to model situations, solve problems, and communicate their thinking.

Selecting Tools and Strategies

Students develop the ability to select appropriate technologies, tools, and strategies to perform particular mathematical tasks, to investigate mathematical ideas, and to solve problems.

Technology. A wide range of technological and digital tools can be used in many contexts for students to interact with, learn, and do mathematics.

Students can use:

- calculators and computers to perform complex operations; create graphs; and collect, organize, and display data;
- digital tools, apps, and social media to investigate mathematical concepts and develop an understanding of mathematical relationships;
- statistical software to manipulate, analyse, represent, sort, and communicate data;
- software to code;
- dynamic geometry software and online geometry tools to develop spatial sense;
- computer programs to represent and simulate mathematical situations (i.e., mathematical modelling);
- communications technologies to support and communicate their thinking and learning;
- computers, tablets, and mobile devices to access mathematical information available on the websites of organizations around the world and to develop information literacy.

Developing the ability to perform mental computations is an important aspect of student learning in mathematics. Students must, therefore, use technology with discretion, and only when it makes sense to do so. When students use technology in their mathematics learning, they should apply mental computation, reasoning, and estimation skills to predict and check answers.

Tools. All students should be encouraged to select and use tools to illustrate mathematical ideas. Students come to understand that making their own representations is a powerful means of building understanding and of explaining their thinking to others. Using tools helps students:

- see patterns and relationships;
- make connections between mathematical concepts and between concrete and abstract representations;
- test, revise, and confirm their reasoning;
- remember how they solved a problem;
- communicate their reasoning to others, including by gesturing.

Strategies. Problem solving often requires students to select an appropriate strategy. Students learn to judge when an exact answer is needed and when an estimate is all that is required, and they use this knowledge to guide their selection. For example, computational strategies include mental computation and estimation to develop a sense of the numbers and operations involved. The selection of a computational strategy is based on the flexibility students have with applying operations to the numbers they are working with. Sometimes, their strategy may involve the use of algorithms or the composition and decomposition of numbers using known facts. Students can also create computational representations of mathematical situations using code.

The Strands in the Mathematics Curriculum

The expectations in the mathematics curriculum are organized into six distinct but related strands: A. Social-Emotional Learning (SEL) Skills in Mathematics and the Mathematical Processes; B. Number; C. Algebra; D. Data; E. Spatial Sense; and F. Financial Literacy.

The program in all grades is designed to ensure that students build a solid foundation in mathematics and develop a positive mathematical identity by connecting and applying mathematical concepts in a variety of ways. To support this process, teachers capitalize on students' prior knowledge, skills, and experiences; integrate concepts from across the strands; and often apply the mathematics that students are learning to types of situations that might occur outside the classroom.

The following chart shows the flow of the learning through the curriculum and the interrelationships among its various components.

Social-Emotional Learning Skills

- Identify and manage emotions
- Recognize sources of stress and cope with challenges
- Maintain positive motivation and perseverance
- Build relationships and communicate effectively
- Develop self-awareness and sense of identity
- Think critically and creatively

Mathematical Processes

- Problem solving
- Reasoning and proving
- Reflecting
- Connecting
- Communicating
- Representing
- Selecting tools and strategies

Strand B. Number	Strand C. Algebra	Strand D. Data	Strand E. Spatial Sense	Strand F. Financial Literacy
B1. Number Sense • whole numbers • rational and irrational numbers • fractions, decimals, and percents **B2. Operations** • properties and relationships • math facts • mental math • addition and subtraction • multiplication and division	**C1. Patterns and Relations** • patterns **C2. Equations and Inequalities** • variables and expressions • equalities and inequalities **C3. Coding** • coding skills **C4. Mathematical Modelling**	**D1. Data Literacy** • data collection and organization • data visualization • data analysis **D2. Probability**	**E1. Geometric and Spatial Reasoning** • geometric reasoning • location and movement **E2. Measurement** • attributes • length • mass, capacity, and volume • area and surface area • angles • time • the metric system	**GRADES 1 TO 8:** **F1. Money** • money concepts **GRADES 4 TO 8:** **F1. Finances** • money concepts • financial management • consumer and civic awareness

Strand A – Social-Emotional Learning (SEL) Skills in Mathematics and the Mathematical Processes

There is strong evidence that developing social-emotional learning skills at school contributes to all students' overall health and well-being and to successful academic performance. It also supports positive mental health, as well as students' ability to learn, build resilience, and thrive. The development of social-emotional learning skills throughout their school years will support all students in becoming healthier and more successful in their daily lives and as contributing members of society. In all grades, learning related to the expectations in this strand occurs in the context of learning related to the other five strands and is assessed and evaluated within these contexts.

Social-emotional learning skills can be developed across all subjects of the curriculum – including mathematics – as well as during various school activities, at home, and in the community. These skills support students in understanding mathematical concepts and in applying the mathematical processes that are key to learning and doing mathematics. They help all students – and indeed all learners, including educators and parents – develop confidence, cope with challenges, and think critically. This in turn enables them to improve and demonstrate mathematics knowledge, concepts, and skills in a variety of situations. Social-emotional learning skills help every student develop a positive identity as a capable "math learner".

In all grades, Strand A comprises a single overall expectation and a chart listing the social-emotional learning skills, the mathematical processes, and the expected outcomes when students use these skills and processes to show their understanding and application of the mathematical content. The progression of learning from grade to grade is indicated in the examples, which are linked to each social-emotional learning skill in each grade and which highlight how the skills can be integrated with learning across the other five strands. The content and application of the learning changes as students develop and mature. *Students' application of the social-emotional learning skills and mathematical processes must be assessed and evaluated as a part of their achievement of the overall expectations in each of the strands for every grade.*

The chart in Strand A outlines the social-emotional learning skills, the mathematical processes, and the expected outcomes when students apply both as they learn and do mathematics. The interaction of skills and processes is variable: Different social-emotional learning skills may be applied at different times in connection with different mathematical processes to achieve the outcomes.

Social-Emotional Learning Skills: Key Components and Sample Strategies

The following chart provides detailed information about each of the skills, including key ideas and sample strategies.

Skills *What are the skills?* *How do they help? What do they look like in mathematics?*	Key Components and Sample Strategies
Identification and Management of Emotions Students often experience a range of emotions over the course of their day at school. They may feel happy, sad, angry, frustrated, or excited, or any number of emotions in combination. Students, and especially younger children, may struggle to identify and appropriately express their feelings. Learning to recognize different emotions, and to manage them effectively, can help students function and interact more successfully. When students understand the influence of thoughts and emotions on behaviour, they can improve the quality of their interactions. In mathematics, as they learn new mathematics concepts and interact with others while problem solving, students have many opportunities to develop awareness of their emotions and to use communication skills to express their feelings and to respond constructively when they recognize emotions in others.	• Recognizing a range of emotions in self and others • Gauging the intensity and/or the level of emotion • Understanding connections between thoughts, feelings, and actions • Recognizing that new or challenging learning may involve a sense of excitement or an initial sense of discomfort • Managing strong emotions and using strategies to self-regulate • Applying strategies such as: ◊ using a "feelings chart" to learn words to express feelings ◊ using a "feelings thermometer" or pictures to gauge intensity of emotion
Stress Management and Coping Every day, students are exposed to a range of challenges that can contribute to feelings of stress. As they learn stress management and coping skills, they come to recognize that stress is a part of life and that it can be managed. We can learn ways to respond to challenges that enable us to "bounce back" and, in this way, build resilience in the face of life's obstacles. Over time, with support, practice, feedback, reflection, and experience, students begin to build a set of personal coping strategies that they can carry with them through life. In mathematics, students work through challenging problems, understanding that their resourcefulness in using coping strategies is helping them build personal resilience.	• Problem solving • Seeking support from peers, teachers, family, or their extended community • Managing stress through physical activity • Applying strategies such as: ◊ breaking a task or problem down into pieces and tackling one piece at a time ◊ thinking of a similar problem ◊ deep breathing ◊ guided imagery ◊ stretching ◊ pausing and reflecting

(continued)

Skills *What are the skills?* *How do they help? What do they look like in mathematics?*	Key Components and Sample Strategies
Positive Motivation and Perseverance Positive motivation and perseverance skills help students to "take a long view" and remain hopeful even when their personal and/or immediate circumstances are difficult. With regular use, practices and habits of mind that promote positive motivation help students approach challenges in life with an optimistic and positive mindset and an understanding that there is struggle in most successes and that repeated effort can lead to success. These practices include noticing strengths and positive aspects of experiences, reframing negative thoughts, expressing gratitude, practising optimism, and practising perseverance – appreciating the value of practice, of making mistakes, and of the learning process. In mathematics students have regular opportunities to apply these practices as they solve problems and develop an appreciation for learning from mistakes as a part of the learning process.	• Reframing negative thoughts and experiences • Practising perseverance • Embracing mistakes as a necessary and helpful part of learning • Reflecting on things to be grateful for and expressing gratitude • Practising optimism • Applying strategies such as: ◊ using an iterative approach by trying out different methods, including estimating and guessing and checking, to support problem solving ◊ supporting peers by encouraging them to keep trying if they make a mistake ◊ using personal affirmations like "I can do this."
Healthy Relationship Skills When students interact in positive and meaningful ways with others, mutually respecting diversity of thought and expression, their sense of belonging within the school and community is enhanced. Learning healthy relationship skills helps students establish positive patterns of communication and inspires healthy, cooperative relationships. These skills include the ability to understand and appreciate another person's perspective, to empathize with others, to listen attentively, to be assertive, and to apply conflict-resolution skills. In mathematics, students have opportunities to develop and practise skills that support positive interaction with others as they work together in small groups or in pairs to solve math problems and confront challenges. Developing these skills helps students to communicate with teachers, peers, and family about mathematics with an appreciation of the beauty and wonder of mathematics.	• Being cooperative and collaborative • Using conflict-resolution skills • Listening attentively • Being respectful • Considering other ideas and perspectives • Practising kindness and empathy • Applying strategies such as: ◊ seeking opportunities to help others ◊ taking turns playing different roles (e.g., leader, scribe or illustrator, data collector, observer) when working in groups

(continued)

Skills *What are the skills?* *How do they help? What do they look like in mathematics?*	Key Components and Sample Strategies
Self-Awareness and Sense of Identity Knowing who we are and having a sense of purpose and meaning in our lives enables us to function in the world as self-aware individuals. Our sense of identity enables us to make choices that support our well-being and allows us to connect with and have a sense of belonging in various cultural and social communities. Educators should note that for First Nations, Métis, and Inuit students, the term "sense of identity and belonging" may also mean belonging to and identifying with a particular community and/or nation. Self-awareness and identity skills help students explore who they are – their strengths, difficulties, preferences, interests, values, and ambitions – and how their social and cultural contexts have influenced them. In mathematics, as they learn new skills, students use self-awareness skills to monitor their progress and identify their strengths; in the process, they build their identity as capable math learners. Educators play a key role in reinforcing that everyone – students, educators, and parents – is a math learner and in sharing an appreciation of the beauty and wonder of mathematics.	• Knowing oneself • Caring for oneself • Having a sense of mattering and of purpose • Identifying personal strengths • Having a sense of belonging and community • Communicating their thinking, positive emotions, and excitement about mathematics • Applying strategies such as: ◊ building their identity as a math learner as they learn independently as a result of their efforts and challenges ◊ monitoring progress in skill development ◊ reflecting on strengths and accomplishments and sharing these with peers or caring adults
Critical and Creative Thinking Critical and creative thinking skills enable us to make informed judgements and decisions on the basis of a clear and full understanding of ideas and situations, and their implications, in a variety of settings and contexts. Students learn to question, interpret, predict, analyse, synthesize, detect bias, and distinguish between alternatives. They practise making connections, setting goals, creating plans, making and evaluating decisions, and analysing and solving problems for which there may be no clearly defined answers. Executive functioning skills – the skills and processes that allow us to take initiative, focus, plan, retain and transfer learning, and determine priorities – also support critical and creative thinking. In all aspects of the mathematics curriculum, students have opportunities to develop critical and creative thinking skills. Students have opportunities to build on prior learning, go deeper, and make personal connections through real-life applications.	• Making connections • Making decisions • Evaluating choices, reflecting on and assessing strategies • Communicating effectively • Managing time • Setting goals • Applying organizational skills • Applying strategies such as: ◊ determining what is known and what needs to be found ◊ using various webs, charts, diagrams, and representations to help identify connections and interrelationships ◊ using organizational strategies and tools, such as planners, trackers, and goal-setting frameworks

Strand B – Number

Understanding how numbers work is foundational to many aspects of mathematics. Recognizing and understanding number properties is foundational to developing an understanding of branches of mathematics such as arithmetic and algebra. In the Number strand, as students progress through Grades 1 to 8, they learn about different types of numbers and how those numbers behave when various operations are applied to them.

A vital aspect of number work in elementary grades is to build what is often called number sense, where students develop the ability to flexibly relate numbers and relate computations. Students who have developed number sense regularly use number relationships to make sense of calculations and to assess the reasonableness of numbers used to describe situations, for example, in the media.

Students learn to count effectively and then become fluent with math facts in order to perform calculations efficiently and accurately, whether mentally or by using algorithms on paper. This strand is built on the belief that it is important to develop automaticity, which is the ability to use mathematical skills or perform mathematical procedures with little or no mental effort. Automaticity with math facts enables students to engage in critical thinking and problem solving.

Most students learn math facts gradually over a number of years, connecting to prior knowledge, using tools and calculators. Mastery comes with practice, and practice helps to build fluency and depth. Students draw on their ability to apply math facts as they manipulate algebraic expressions, equations, and inequalities. Mental math skills involve the ability to perform mathematical calculations without relying on pencil and paper. They enable students to estimate answers to calculations, and so to work accurately and efficiently on everyday problems and judge the reasonableness of answers that they have arrived at through calculation. In order to develop effective mental math strategies, all students need to have strong skills in number sense and a solid conceptual understanding of the operations.

Though individual students may progress at different rates, generally speaking, addition/subtraction facts should be mastered by the end of Grade 3, and multiplication/division facts should be mastered by the end of Grade 5.[3] However, all students should continue to learn about effective strategies and to practice and extend their proficiency in the operations throughout the grades and in the context of learning in all the strands of the mathematics curriculum.

3. Chapin, S. H., & Johnson, A. (2006). *Math matters: Understanding the math you teach, Grades K–8* (2nd ed.). Sausalito, CA: Math Solutions Publications.

Strand C – Algebra

In this strand, students develop algebraic reasoning through working with patterns, variables, expressions, equations, inequalities, coding, and the process of mathematical modelling.

As students progress through the grades, they study a variety of patterns, including those found in real life. Students learn to identify regularities in numeric and non-numeric patterns and classify them based on the characteristics of those regularities. They create and translate patterns using various representations. Students determine pattern rules for various patterns in order to extend them, make near and far predictions, and determine their missing elements. They develop recursive and functional thinking as well as additive and multiplicative thinking as they work with linear patterns, and use this thinking to develop the general terms of the patterns to solve for unknown values. Understanding patterns and determining the relationship between two variables has many connections to science and is foundational to further mathematics. In the primary grades, students focus on understanding which quantities remain the same and which can change in everyday contexts, and on how to establish equality between numerical expressions. In the junior and intermediate grades, students work with variables in algebraic expressions, equations, and inequalities in various contexts.

As students progress through the grades, their coding experiences also progress, from representing movements on a grid, to solving problems involving optimization, to manipulating models to find which one best fits the data they are working with in order to make predictions. Coding can be incorporated across all strands and provides students with opportunities to apply and extend their math thinking, reasoning, and communicating.

Students in all grades also engage in the process of mathematical modelling.

The Mathematical Modelling Process

Mathematical modelling provides authentic connections to real-life situations. The process starts with ill-defined, often messy real-life problems that may have several different solutions that are all correct. Mathematical modelling requires the modeller to be critical and creative and make choices, assumptions, and decisions. Through this process, they create a mathematical model that describes a situation using mathematical concepts and language, and that can be used to solve a problem or make decisions and can be used to deepen understanding of mathematical concepts.

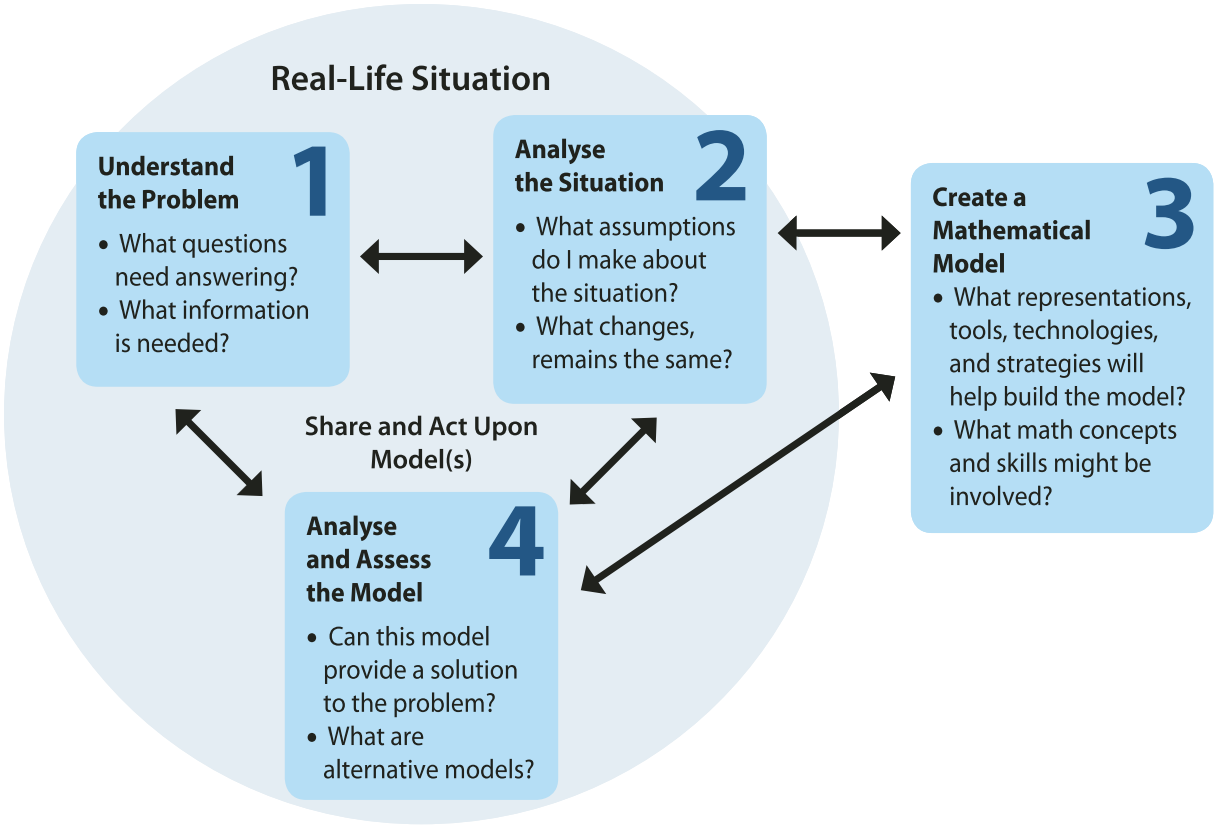
The process of mathematical modelling[4] has four key components that are interconnected and applied in an iterative way, where students may move between and across, as well as

4. Hirsch, C. R., & McDuffie, A. R. (Eds.). (2016). *Annual perspectives in mathematics education 2016: Mathematical modeling and modeling mathematics*. Reston, VA: National Council of Teachers of Mathematics.

return to, each of the four components as they change conditions to observe new outcomes until the model is ready to be shared and acted upon. While moving through these components, social-emotional learning skills and mathematical processes are applied as needed.

1. Understand the problem
   - ◇ What questions need answering?
   - ◇ What information is needed?
2. Analyse the situation
   - ◇ What assumptions do I make about the situation?
   - ◇ What changes, what remains the same?
3. Create a mathematical model
   - ◇ What representations, tools, technologies, and strategies will help build the model?
   - ◇ What mathematical knowledge, concepts, and skills might be involved?
4. Analyse and assess the model
   - ◇ Can this model provide a solution?
   - ◇ What are alternative models?

The Process of Mathematical Modelling



Strand D – Data

The related topics of statistics and probability, which are addressed in this strand, are highly relevant to real life. The public is bombarded with data through advertising, opinion polls, politics, population trends, and scientific discoveries, to name just a few. Thus, one of the key focuses in this strand is to support students in developing critical thinking skills throughout their development of data literacy, so that they can analyse, synthesize, understand, generate, and use data, both as consumers and producers.

The main purpose for collecting and organizing data is to gather information in order to answer questions. When questions stimulate students' curiosity, they become engaged in collecting, organizing, and interpreting the data that provide answers to their questions. Relevant questions often arise from class discussions; classroom events, issues, and thematic activities; and topics in various subject areas. When students collect and organize data, they have an opportunity to learn more about themselves, their environment, issues in their school or community, and so on. Learning activities should help students understand the processes that are involved in formulating questions, seeking relevant information, and organizing that information in meaningful ways. Involving students in collecting and organizing data allows them to participate in the decision making that is required at different steps of the process.

As students progress through the grades, they develop an understanding of qualitative data and both discrete and continuous quantitative data, and use that understanding to select appropriate ways to organize and display data. Students learn the fundamentals of statistics and develop the skills to visualize and critically analyse data, including identifying any possible biases within the data. Starting in the junior grades, students make intentional choices in creating infographics in order to represent key information about a set of data for a particular audience and to engage in the critical interpretation of data. In addition, students learn how to use data to make compelling arguments about questions of interest.

The learning in this strand also supports students in developing probabilistic reasoning. As students progress through the grades, they begin to understand the relationship between probability and data, and how data is used to make predictions about populations. Students' intuitive understanding of probability is nurtured in the early grades to help them make connections to their prior experience with probability in everyday life, beginning with simply understanding that some events are likely to happen while others are not likely. Eventually, students begin to understand and represent these probabilities as fractions, decimals and percents. From Grades 5 to 8, students compare experimental probabilities involving independent and dependent events with their theoretical probabilities, and use these measures to make predictions about events.

Strand E – Spatial Sense

This strand combines the areas of geometry and measurement in order to emphasize the relationship between the two areas and to highlight the role of spatial reasoning in underpinning the development of both. Study in this strand provides students with the language and tools to analyse, compare, describe, and navigate the world around them. It is a gateway to professions in other STEM (science, technology, engineering, and mathematics) disciplines, and builds foundational skills needed for construction, architecture, engineering, research, and design.

In this strand, students analyse the properties of shapes – the elements that define a shape and make it unique – and use these properties to define, compare, and construct shapes and objects, as well as to explore relationships among properties. Students begin with an intuition about their surroundings and the objects in them, and learn to visualize objects from different perspectives. Over time, students develop an increasingly sophisticated understanding of size, shape, location, movement, and change, in both two and three dimensions. They understand and choose appropriate units to estimate, measure, and compare attributes, and they use appropriate tools to make measurements. They apply their understanding of the relationships between shapes and measurement to develop formulas to calculate length, area, volume, and more.

Strand F – Financial Literacy

All Ontario students need the skills and knowledge to take responsibility for managing their personal financial well-being with confidence, competence, a critical and compassionate awareness of the world around them.

Financial Literacy is a dedicated strand throughout the elementary math curriculum. Financial literacy is more than just knowing about money and financial matters and having the skills to work with this knowledge. Students develop the confidence and capacity to successfully apply the necessary knowledge, concepts, and skills in a range of relevant real-life contexts and for a range of purposes. They also develop the ability to make informed decisions as consumers and citizens while taking into account the ethical, societal, environmental, and personal aspects of those decisions.

In Grades 1 to 3, students demonstrate an understanding of the value and use of money by recognizing Canadian coins and bills, representing various amounts, and calculating change in simple transactions. In Grades 4 to 8, students extend their learning to the knowledge, concepts, and skills required to make informed financial decisions relevant to their lived experiences and plan simple sample budgets. Students begin to develop consumer and civic awareness in the junior and intermediate grades. Making connections to what they are learning in the Media Literacy strand of the language curriculum as well as the social studies, history and geography curriculum, students become informed consumers and learn about the broader economic systems in their local communities, communities in other global contexts that their families are connected to, and beyond. Educators consider and respond to the range of equity issues related to the diverse circumstances and lived experiences of students and their families.

This strand connects with other mathematics strands in many ways, such as applying knowledge, concepts, and skills related to:

- numbers and operations to calculate change;
- percents to calculate sales tax and interest;
- mathematical modelling to understand real-life financial situations, including the financial applications of linear rates;
- unit rates to compare goods and services, and mental math to quickly determine those with the best value;
- social-emotional learning to become confident and critical consumers, and to persevere in managing financial well-being.

Some Considerations for Program Planning in Mathematics

Teachers consider many factors when planning a mathematics program that cultivates the best possible environment in which all students can maximize their mathematical learning. This section highlights the key strategies and approaches that teachers and school leaders should consider as they plan effective and inclusive mathematics programs. Additional information can be found in the "Considerations for Program Planning" section, which provides information applicable to all curricula.

Instructional Approaches in Mathematics

Instruction in mathematics should support all students in acquiring the knowledge, skills, and habits of mind they need in order to achieve the curriculum expectations and be able to enjoy and participate in mathematics learning for years to come.

Effective math instruction begins with knowing the identity and profile of the students, having high expectations for and of all students, and believing that all students can learn and do mathematics. It uses culturally relevant and responsive practices and differentiated learning experiences to meet individual students' learning strengths and needs. It focuses on the development of conceptual understanding and procedural fluency, skill development, communication, and problem-solving skills. It takes place in a safe, positive, and inclusive learning environment, where all students feel valued, empowered, engaged, and able to take risks and approach the learning of mathematics in a confident manner. Instruction that is student-centred and that builds on students' strengths is effective as it motivates and engages students meaningfully and instils positive habits of mind, such as curiosity and open-mindedness; a willingness to think, to question, to challenge and be challenged; and an awareness of the value of listening intently, reading thoughtfully, and communicating with clarity.

Learning should be relevant and inspired by the lived realities of all students and embedded in authentic, real-life contexts that allow students to develop the fundamental mathematical concepts and skills and to see the beauty and wide-ranging nature of mathematics. This approach enables students to use mathematical reasoning to see connections throughout their lives.

High-Impact Practices

Teachers understand the importance of knowing the identities and profiles of all students and of choosing the instructional approaches that will best support student learning. The approaches that teachers employ vary according to both the learning outcomes and the needs of the students, and teachers choose from and use a variety of accessible, equitable high-impact instructional practices.

The thoughtful use of these high-impact instructional practices – including knowing when to use them and how they might be combined to best support the achievement of specific math goals – is an essential component of effective math instruction. Researchers have found that the following practices consistently have a high impact on teaching and learning mathematics:[5]

- **Learning Goals, Success Criteria, and Descriptive Feedback.** Learning goals and success criteria outline the intention for the lesson and how this intention will be achieved to ensure teachers and students have a clear and common understanding of what is being learned and what success looks like. The use of descriptive feedback involves providing students with the precise information they need in order to reach the intended learning goal. Using this practice makes all other practices more effective.
- **Direct Instruction.** This is a concise, intentional form of instruction that begins with a clear learning goal. It is not a lecture or a show-and-tell. Instead, direct instruction is a carefully planned and focused approach that uses questioning, activities, or brief demonstrations to guide learning, check for understanding, and make concepts clear. Direct instruction prioritizes feedback and formative assessment throughout the learning process and concludes with a clear summary of the learning.
- **Problem-Solving Tasks and Experiences.** It is an effective practice to use a problem, carefully chosen by the teacher or students, to introduce, clarify, or apply a concept or skill. This practice provides opportunities for students to demonstrate their agency by representing, connecting, and justifying their thinking. Students communicate and reason with one another and generate ideas that the teacher connects in order to highlight important concepts, refine existing understanding, eliminate unfruitful strategies, and advance learning.
- **Teaching about Problem Solving.** Teaching students about the process of problem solving makes explicit the critical thinking that problem solving requires. It involves teaching students to identify what is known and unknown and to draw on similarities between various types of problems. Teaching about problem solving involves using representations to model the problem-solving situation. This practice reinforces that

5. Hattie, J., Fisher, D., Frey, N., Gojak, L. M., Moore, S. D., & Mellman, W. (2017). *Visible learning for mathematics: What works best to optimize student learning, Grades K–12*. Thousand Oaks, CA: Corwin Mathematics.

problem solving requires perseverance and an awareness that mistakes can ultimately lead to growth in learning.

- **Tools and Representations.** The use of a variety of appropriate tools and representations supports a conceptual understanding of mathematics at all grade levels. Carefully chosen and used effectively, representations and tools such as manipulatives make math concepts accessible to a wide range of learners. At the same time, student interactions with representations and tools also give teachers insight into students' thinking and learning.
- **Math Conversations.** Effective math conversations provide multiple opportunities for all students to engage in meaningful math talk by listening to and responding to the ideas of others. These conversations involve reasoning, proving, building on the thinking of others, defending and justifying their own thinking, and adjusting their perspectives as they build their mathematical understanding, confidence, and identity.
- **Small-Group Instruction.** A powerful strategy for moving student learning forward, small-group instruction involves targeted and timely mathematics instruction that meets the learning needs of specific students at appropriate times. By working with small and flexible groups, whether they are homogenous or heterogenous, the teachers can personalize learning in order to prevent gaps from developing, close gaps that already exist, or extend thinking. Small-group instruction also provides opportunities for teachers to learn more about student identities, experiences, and communities, which the teachers can use as a basis for their mathematics instruction.
- **Deliberate Practice.** Practice is best when it is purposeful and spaced over time. It must always follow understanding. This ensures that there is continual, consistent, and relevant feedback, so students know that they are practising correctly. Practice is an essential part of an effective mathematics program.
- **Flexible Groupings.** The intentional combination of large-group, small-group, partnered, and independent working arrangements, in response to student and class learning needs, can foster a rich mathematical learning environment. Creating flexible groupings in a mathematics class enables students to work independently of the teacher but with the support of their peers, and it strengthens collaboration and communication skills. Regardless of the size of the group, it is of utmost importance that individual students are accountable for and have ownership of their learning.

While a lesson may prominently feature one of these high-impact practices, other practices will inevitably also be involved. The practices are rarely used in isolation, nor is there any single "best" instructional practice. Teachers choose the right practice, for the right time, in order to create an optimal learning experience for all students. They use their knowledge of the students, a deep understanding of the curriculum and of the mathematics that

underpins the expectations, and a variety of assessment strategies to determine which high-impact instructional practice, or combination of practices, best supports the students. These decisions are made continually throughout a lesson. The appropriate use of high-impact practices plays an important role in supporting student learning.

The Role of Information and Communication Technology in Mathematics

The mathematics curriculum was developed with the understanding that the strategic use of technology is part of a balanced mathematics program. Technology can extend and enrich teachers' instructional strategies to support all students' learning in mathematics. Technology, when used in a thoughtful manner, can support and foster the development of mathematical reasoning, problem solving, and communication.

When using technology to support the teaching and learning of mathematics, teachers consider the issues of student safety, privacy, ethical responsibility, equity and inclusion, and well-being.

The strategic use of technology to support the achievement of the curriculum expectations requires a strong understanding of:

- the mathematical concepts being addressed;
- high-impact teaching practices that can be used as appropriate to achieve the learning goals;
- the capacity of the chosen technology to augment the learning, and how to use this technology.

Technology can be used specifically to support the "doing" of mathematics (e.g., digital tools, computation devices, calculators, data-collection programs) or to facilitate access to information and allow better communication and collaboration (e.g., collaborative documents and web-based content that enable students to connect with experts and other students, near or far). Technology can support English language learners in accessing mathematics terminology and ways of solving problems in their first language. Assistive technologies are critical in enabling some students with special education needs to have equitable access to the curriculum and in supporting their learning, and must be provided in accordance with students' Individual Education Plan (IEP), as required.

Teachers understand the importance of technology and how it can be leveraged to support learning and to ensure that the mathematics curriculum expectations can be met by all students. Additional information can be found in the "The Role of Information and Communications Technology" subsection of "Considerations for Program Planning".

Planning Mathematics Programs for Students with Special Education Needs

Classroom teachers are the key educators of students with special education needs. They have a responsibility to support all students in their learning, and they work collaboratively with special education teachers, where appropriate, to achieve this goal. Classroom teachers commit to assisting every student to prepare for living with the highest degree of independence possible. More information on planning for and assessing students with special education needs can be found in the "Planning for Students with Special Education Needs" subsection of "Considerations for Program Planning".

Principles for Supporting Students with Special Education Needs

The following principles[6] guide teachers in effectively planning and teaching mathematics programs to students with special education needs, and also benefit all students:

- The teacher plays a critical role in student success in mathematics.
- It is important for teachers to develop an understanding of the general principles of how students learn mathematics.
- The learning expectations outline developmentally appropriate key concepts and skills of mathematics across all of the strands that are interconnected and foundational.
- There is an important connection between procedural knowledge and conceptual understanding of mathematics.
- The use of concrete representations and tools is fundamental to learning mathematics in all grades and provides a way of representing both concepts and student understanding.
- The teaching and learning process involves ongoing assessment. Students with special education needs should be provided with various opportunities to demonstrate their learning and thinking in multiple ways.

An effective mathematics learning environment and program that addresses the mathematical learning needs of students with special education needs is purposefully planned with the principles of Universal Design for Learning in mind and integrates the following elements:

- knowing the student's strengths, interests, motivations, and needs in mathematics learning in order to differentiate learning and make accommodations and modifications as outlined in the student's Individual Education Plan;
- building the student's confidence and a positive identity as a mathematics learner;

6. Adapted from Ontario. Ministry of Education. (2005) *Education for All: The Report of the Expert Panel on Literacy and Numeracy Instruction for Students with Special Education Needs, Kindergarten to Grade 6*. Toronto: Author.

- valuing the student's prior knowledge and connecting what the student knows with what the student needs to learn;
- focusing on the connections between broad concepts in mathematics;
- connecting mathematics with familiar, relevant, everyday situations and providing rich and meaningful learning contexts;
- fostering a positive attitude towards mathematics and an appreciation of mathematics through multimodal means, including through the use of assistive technology and the performance of authentic tasks;
- implementing research-informed instructional approaches (e.g., Concrete – Semi-Concrete – Representational – Abstract) when introducing new concepts to promote conceptual understanding, procedural accuracy, and fluency;
- creating a balance of explicit instruction, learning in flexible groupings, and independent learning. Each form of learning should take place in a safe, supportive, and stimulating environment while taking into consideration that some students may require more systematic and intensive support, more explicit and direction instruction before engaging in independent learning;
- providing environmental, assessment, and instructional accommodations that are specified in the student's Individual Education Plan in order to maximize the student's learning (e.g., making available learning tools such as manipulatives, resources, adapted game pieces, oversized tangrams, and calculators; ensuring access to assistive technology);
- building an inclusive community of learners and encouraging students with special education needs to participate in various mathematics-oriented class projects and activities;
- building partnerships with administrators and other teachers, particularly special education teachers, where available, to share expertise and knowledge of the curriculum expectations; co-develop content in the Individual Education Plan that is specific to mathematics; and systematically implement intervention strategies, as required, while making meaningful connections between school and home to ensure that what the student is learning in the school is relevant and can be practised and reinforced beyond the classroom.

Planning Mathematics Programs for English Language Learners

English language learners are working to achieve the curriculum expectations in mathematics while they are acquiring English-language proficiency. An effective mathematics program that supports the success of English language learners is purposefully planned with the following considerations in mind.

- Students' various linguistic identities are viewed as a critical resource in mathematics instruction and learning. This enables students to use their linguistic repertoire in a fluid and dynamic way, mixing and meshing languages to communicate. This translingual practice[7] is creative and strategic, and allows students to communicate, interact, and connect with peers and teachers for a variety of purposes, such as when developing conceptual knowledge and when seeking clarity and understanding.
- Knowledge of English language learners' mathematical strengths, interests, and identities, including their social and cultural backgrounds is important. These "funds of knowledge"[8] are historically and culturally developed skills and assets that can be incorporated into mathematics learning to create a richer and more highly scaffolded learning experience for all students, promoting a positive, inclusive teaching and learning environment.
- In addition to assessing their level of English-language proficiency, an initial assessment of the math knowledge and skills of newcomer English language learners is required in Ontario schools.
- Differentiated instruction is essential in supporting English language learners, who face the dual challenge of learning new conceptual knowledge while acquiring English-language proficiency. Designing mathematics learning to have the right balance for English language learners is achieved through program adaptations (i.e., accommodations and/or modifications) that ensure the tasks are mathematically challenging, reflective of learning demands within the mathematics curriculum, and comprehensible and accessible to English language learners. Using the full range of a student's language assets, including additional languages that a student speaks, reads, and writes, as a resource in the mathematics classroom supports access to their prior learning, reduces the language demands of the mathematics curriculum, and increases engagement;
- Working with students and their families and with available community supports allows for the multilingual representation of mathematics concepts to create relevant and real-life learning contexts and tasks.

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7. García, O., Johnson, S. I., & Seltzer, K. (2017). *The translanguaging classroom: Leveraging student bilingualism for learning*. Philadelphia, PA: Caslon.

8. Marshall, E., & Toohey, K. (2010). Representing family: Community funds of knowledge, bilingualism, and multimodality. *Harvard Educational Review, 80*(2), 221-242.

In a supportive learning environment, scaffolding the learning of mathematics assessment and instruction offers English language learners the opportunity to:

- access their other language(s) (e.g., by using technology to access mathematical terminology and ways of solving problems in their first language), prior learning experiences, and background knowledge in mathematics;
- learn new mathematical concepts in authentic, meaningful, and familiar contexts;
- engage in open and parallel tasks to allow for multiple entry points for learning;
- work in a variety of settings that support co-learning and multiple opportunities to practice (e.g., with partners or in small groups, as part of cooperative learning, in group conferences);
- access the language of instruction during oral, written, and multimodal instruction and assessment, during questioning, and when encountering texts, learning tasks, and other activities in the mathematics program;
- use oral language in different strategically planned activities, such as "think-pair-share", "turn-and-talk", and "adding on", to express their ideas and engage in mathematical discourse;
- develop both everyday and academic vocabulary, including specialized mathematics vocabulary in context, through rephrasing and recasting by the teacher and through using student-developed bilingual word banks or glossaries;
- practise using sentence frames adapted to their English-language proficiency levels to describe concepts, provide reasoning, hypothesize, make judgements, and explain their thinking;
- use a variety of concrete and/or digital learning tools to demonstrate their learning in mathematics in multiple ways (e.g., orally, visually, kinesthetically), through a range of representations (e.g., portfolios, displays, discussions, models), and in multiple languages (e.g., multilingual word walls and anchor charts);
- have their learning assessed in terms of the processes they use in multiple languages, both during the learning and through teachers' observations and conversations.

Strategies used to differentiate instruction and assessment for English language learners in the mathematics classroom also benefit many other learners in the classroom, since programming is focused on leveraging all students' strengths, meeting learners where they are in their learning, being aware of language demands in the mathematics program, and making learning visible. For example, different cultural approaches to solve mathematical problems can help students make connections to the Ontario curriculum and provide classmates with alternate ways of solving problems.

English language learners in English Literacy Development (ELD) programs in Grades 3 to 8 require accelerated support to develop both their literacy skills and their numeracy skills. These students have significant gaps in their education because of limited or

interrupted prior schooling. They are learning key mathematical concepts missed in prior years in order to build a solid foundation of mathematics. At the same time, they are learning the academic language of mathematics in English while not having acquired developmentally appropriate literacy skills in their first language. Programming for these students is, therefore, highly differentiated and intensive. These students often require focused support over a longer period than students in English as a Second Language (ESL) programs. The use of the student's oral competence in languages other than English is a non-negotiable scaffold. The strategies described above, such as the use of visuals, the development of everyday and academic vocabulary, the use of technology, and the use of oral competence, are essential in supporting student success in ELD programs and in mathematics.

Supporting English language learners is a shared responsibility. Collaboration with administrators and other teachers, particularly ESL/ELD teachers, where possible, contributes to creating equitable outcomes for English language learners. Additional information on planning for and assessing English language learners can be found in the "Planning for English Language Learners" subsection of "Considerations for Program Planning".

Human Rights, Equity, and Inclusive Education in Mathematics

Research indicates that there are groups of students who continue to experience systemic barriers to learning mathematics. Systemic barriers can result in inequitable outcomes, such as chronic underachievement and low confidence in mathematics. Achieving equitable outcomes in mathematics for all students requires educators to pay attention to these barriers and to how they can overlap and intersect, compounding their effect. Educators ensure that students have access to enrichment support, as necessary, and they capitalize on the rich cultural knowledge, experience, and competencies that all students bring to mathematics learning. When educators develop pedagogical practices that are differentiated, culturally relevant, and responsive, and hold high and appropriate expectations of students, they maximize the opportunity for all students to learn, and they create the conditions necessary to ensure that students have a positive identity as a mathematics learner and can succeed in mathematics and in all other subjects.

It is essential to develop practices that learn from and build on students' cultural competencies and linguistic resources, recognizing that students bring a wealth of mathematical knowledge, information, experiences, and skills into the classroom, often in languages different from the language of instruction. Educators create the conditions for authentic mathematics experiences by connecting mathematics learning to students' communities and lives; by respecting and harnessing students' prior knowledge, experiences, strengths, and interests; and by acknowledging and actively reducing and eliminating the systemic

barriers that some students face. Mathematics learning that is student-centered allows students to find relevance and meaning in what they are learning, to make real-life connections to the curriculum.

Mathematics classrooms also provide an opportunity for cross-curricular learning and for teaching about human rights. To create safe, inclusive, and engaging learning environments, educators must be committed to equity and inclusion for all students and to upholding and promoting human rights. Every student, regardless of their background, identity, or personal circumstances, has the right to have mathematics opportunities that allow them to succeed, personally and academically. In any mathematics classroom, it is crucial to acknowledge students' multiple social identities and how students intersect with the world. Educators have an obligation to develop and nurture learning environments that are reflective of and responsive to students' strengths, needs, cultures, and diverse lived experiences, and to set appropriate and high expectations for all.

Culturally Relevant and Responsive Pedagogy in Mathematics

Rich, high-quality instruction and tasks are the foundation of culturally relevant and responsive pedagogy (CRRP) in mathematics. In CRRP classrooms, teachers learn about their own identities and pay attention to how those identities affect their teaching, their ideas, and their biases. Teachers also learn about students' identities, identifications, and/or affiliations and build on students' ideas, questions, and interests to support the development of an engaging mathematics classroom community.

In mathematics spaces using CRRP, students are engaged in shaping much of the learning so that students have mathematical agency and feel invested in the outcomes. Students develop agency that motivates them to take ownership of their learning of, and progress in, mathematics. Teaching about diverse mathematical figures in history and from different global contexts enables students not only to see themselves reflected in mathematical learning – a key factor in developing students' sense of self – but also to learn about others, and the multiple ways mathematics exists in all aspects of the world around them.

Culturally reflective and responsive teachers know that there is more than one way to develop a solution. Students are exposed to multiple ways of knowing and are encouraged to explore multiple ways of finding answers. For example, an Indigenous pedagogical approach emphasizes holistic, experiential learning; teacher modelling; and the use of collaborative and engaging activities. Teachers differentiate instruction and assessment opportunities to encourage different ways of learning, to allow all students to learn from and with each other, and to promote an awareness of and respect for the diverse and multiple ways of knowing that make up our classrooms, schools, and the world. When making connections between mathematics and real-life applications, teachers may work in partnership with Indigenous communities to co-teach. Teachers may respectfully

incorporate Indigenous culturally specific examples as a way to meaningfully infuse Indigenous knowledge into the mathematics program. In this way, culturally specific examples can be used without cultural appropriation.

More information on equity and inclusive education can be found in the "Human Rights, Equity, and Inclusive Education" subsection of "Considerations for Program Planning".

Cross-Curricular and Integrated Learning in Mathematics

When planning an integrated mathematics program, educators should consider that, although the mathematical content in the curriculum is outlined in discrete strands, students develop mathematical thinking, such as proportional reasoning, algebraic reasoning, and spatial reasoning, that transcends the expectations in the strands and even connects to the learning in many other subject areas. By purposefully drawing connections across all areas of mathematics and to other subject areas, and by applying learning to relevant real-life contexts, teachers extend and enhance student learning experiences and deepen their knowledge and skills across disciplines and beyond the classroom.

For example, proportional reasoning, which is developed through the study of ratios and rates in the Number strand, is also used when students are working towards meeting learning expectations in other strands of the math curriculum, such as Spatial Sense, and in other disciplines, such as science, geography, and the arts. Students then apply this learning in their everyday lives – for example, when adjusting a recipe or preparing a mixture or solutions.

Similarly, algebraic reasoning is applied beyond the Algebra strand. For example, it is applied in measurement when learning about formulas, such as *area of a parallelogram* = *base* × *height*. It is applied in other disciplines, such as science, when students study simple machines and learn about the formula *work* = *force* × *distance*. Algebraic reasoning is also used when making decisions in everyday life – for example, when determining which service provider offers a better consumer contract or when calculating how much time it will take for a frozen package to thaw.

Spatial thinking has a fundamental role throughout the Ontario curriculum, from Kindergarten to Grade 12, including in mathematics, the arts, health and physical education, and science. For example, a student demonstrates spatial reasoning when mentally rotating and matching shapes in mathematics, planning their move to the basketball hoop, and using diagonal and converging lines to create perspective drawing in visual art. In everyday life, there are many applications of spatial reasoning, such as when creating a garden layout or when using a map to navigate the most efficient way of getting from point A to point B.

Teaching mathematics as a narrowly defined subject area places limits on the depth of learning that can occur. When teachers work together to develop integrated learning opportunities and highlight cross-curricular connections, students are better able to:

- make connections between mathematics and other subject areas, and among the strands of the mathematics curriculum;
- improve their ability to provide multiple responses to a problem;
- debate and test whether responses are effective and efficient;
- apply a range of knowledge and skills to solve problems in mathematics and in their daily experiences and lives.

When students are provided with opportunities to learn mathematics through real-life applications, integrating learning expectations from across the curriculum, they use their knowledge of other subject matter to enhance their learning of and engagement in mathematics. More information about integrating learning across the curriculum can be found in "Cross-Curricular and Integrated Learning".

Literacy in Mathematics

Literacy skills needed for reading and writing in general are essential for the learning of mathematics. To engage in mathematical activities and develop computational fluency, students require the ability to read and write mathematical expressions, to use a variety of literacy strategies to comprehend mathematical text, to use language to analyse, summarize, and record their observations, and to explain their reasoning when solving problems. Research shows that "mathematics texts contain more concepts per sentence and paragraph than any other type of text".[9] Reading a mathematics text requires specific literacy strategies, unique to mathematics. Learning in "some areas of math in particular, such as word problems and number combinations may be mediated by language and reading due to the nature of the task."[10] As a result, there is a strong correlation between reading and math achievement.

The learning of mathematics requires students to navigate discipline-specific texts that "must be written and read in appropriate ways"; therefore, it is important that math instruction addresses both "mathematical texts and literacies."[11] Many of the activities and tasks students undertake in mathematics involve the use of written, oral, visual, and

9. Kenney, Joan M. et al. (2007). *Literacy strategies for improving mathematics instruction*. Heatherton, Vic: Hawker Brownlow Education. Page 11.

10. Rutherford-Becker, Kristy J. & Vanderwood, Michael L. (2009). Evaluation of the relationship between literacy and mathematics skills as assessed by curriculum-based measures. *California School Psychologist*, Vol. 14, page 25.

11. Siebert, D., & Hendrickson, S. (2010). (Re)imagining literacies for mathematics classrooms. In R. J. Draper, P. Broomhead, A. P. Jensen, J. D. Nokes, & D. Siebert (Eds.), *(Re)imagining content-area literacy instruction*. New York, NY: Teachers College Press. Page 43.

multimodal communication skills as they encounter mathematical texts such as "equations, graphs, diagrams, proofs, justifications, displays of manipulatives (e.g., base ten blocks), calculator readouts, verbal mathematical discussions, and written descriptions of problems."[12] The language of mathematics includes special terminology. To support all students in developing an understanding of mathematical texts, teachers need to explicitly teach mathematical vocabulary, focusing on the many meanings and applications of the terms students may encounter. In all mathematics programs, students are required to use appropriate and correct terminology and are encouraged to use language with care and precision in order to communicate effectively.

More information about the importance of literacy across the curriculum can be found in the "Literacy" and "Mathematical Literacy" subsections of "Cross-curricular and Integrated Learning".

Transferable Skills in Mathematics

The Ontario curriculum emphasizes a set of skills that are critical to all students' ability to thrive in school, in the world beyond school, and in the future. These are known as transferable skills. Educators facilitate students' development of transferable skills across the curriculum, from Kindergarten to Grade 12. They are as follows:

- **Critical Thinking and Problem Solving.** In mathematics, students and educators learn and apply strategies to understand and solve problems flexibly, accurately, and efficiently. They learn to understand and visualize a situation and use the tools and language of mathematics to reason, make connections to real-life situations, communicate, and justify solutions.
- **Innovation, Creativity, and Entrepreneurship.** In mathematics, students and educators solve problems with curiosity, creativity, and a willingness to take risks. They pose questions, make and test conjectures, and consider problems from different perspectives to generate new learning and apply it to novel situations.
- **Self-Directed Learning.** By reflecting on their own thinking and emotions, students, with the support of educators, can develop perseverance, resourcefulness, resilience, and a sense of self. In mathematics, they initiate new learning, monitor their thinking and their emotions when solving problems and apply strategies to overcome challenges. They see mathematics as useful, interesting, and doable and confidently look for ways to apply their learning.

12. Siebert, D., & Hendrickson, S. (2010). (Re)imagining literacies for mathematics classrooms. In R. J. Draper, P. Broomhead, A. P. Jensen, J. D. Nokes, & D. Siebert (Eds.), *(Re)imagining content-area literacy instruction*. New York, NY: Teachers College Press. Page 41.

- **Collaboration.** In mathematics, students and educators engage with others productively, respectfully, and critically in order to better understand ideas and problems, generate solutions, and refine their thinking.
- **Communication.** In mathematics, students and educators use the tools and language of mathematics to describe their thinking and to understand the world. They use mathematical vocabulary, symbols, conventions, and representations to make meaning, express a point of view, and make convincing and compelling arguments in a variety of ways, including multimodally, for example, using combinations of oral, visual, textual, and gestural communication.
- **Global Citizenship and Sustainability.** In mathematics, students and educators recognize and appreciate multiple ways of knowing, doing, and learning, and value different perspectives. They see how mathematics is used in all walks of life and how engaged citizens can use it as a tool to raise awareness and generate solutions for real-life issues.
- **Digital Literacy.** In mathematics, students and educators learn to be discerning users of technology. They select when and how to use appropriate tools to understand and model real-life situations, predict outcomes, and solve problems, and they assess and evaluate the reasonableness of their results.

More information on instructional approaches can be found in the "Transferable Skills" section of "Program Planning".

Assessment and Evaluation of Student Achievement

Growing Success: Assessment, Evaluation, and Reporting in Ontario Schools, First Edition, Covering Grades 1 to 12, 2010 sets out the Ministry of Education's assessment, evaluation, and reporting policy. The policy aims to maintain high standards, improve student learning, and benefit all students, parents, and teachers in elementary and secondary schools across the province. Successful implementation of this policy depends on the professional judgement[13] of teachers at all levels as well as their high expectations of all students, and on their ability to work together and to build trust and confidence among parents and students.

Major aspects of assessment, evaluation, and reporting policy are summarized in the main "Assessment and Evaluation" section. The key tool for assessment and evaluation in mathematics – the achievement chart – is provided below.

13. Professional judgement", as defined in *Growing Success* (p. 152), is "judgement that is informed by professional knowledge of curriculum expectations, context, evidence of learning, methods of instruction and assessment, and the criteria and standards that indicate success in student learning. In professional practice, judgement involves a purposeful and systematic thinking process that evolves in terms of accuracy and insight with ongoing reflection and self-correction.

The Achievement Chart for Mathematics

The achievement chart identifies four categories of knowledge and skills and four levels of achievement in mathematics. (For important background, see "Content Standards and Performance Standards" in the main Assessment and Evaluation section.)

Knowledge and Understanding – Subject-specific content acquired in each grade (knowledge), and the comprehension of its meaning and significance (understanding)

Categories	Level 1	Level 2	Level 3	Level 4
	The student:			
Knowledge of content *(e.g., math facts, computational strategies, terminology, mathematical models, money values)*	demonstrates limited knowledge of content	demonstrates some knowledge of content	demonstrates considerable knowledge of content	demonstrates thorough knowledge of content
Understanding of content *(e.g., concepts, theories, procedures, principles, mathematical processes)*	demonstrates limited understanding of content	demonstrates some understanding of content	demonstrates considerable understanding of content	demonstrates thorough understanding of content

Thinking – The use of critical and creative thinking skills and/or processes

Categories	Level 1	Level 2	Level 3	Level 4
	The student:			
Use of planning skills *(e.g., interpreting and expressing problems, identifying unknown(s), making conjectures and estimates, identifying steps to take, considering the use of models and representations, selecting strategies and tools)*	uses planning skills with limited effectiveness	uses planning skills with some effectiveness	uses planning skills with considerable effectiveness	uses planning skills with a high degree of effectiveness
Use of processing skills* *(e.g., carrying out plans: collecting data, questioning, testing, revising, modelling, solving, inferring, forming conclusions; looking back at solutions: reflecting, evaluating reasonableness, reasoning, justifying, proving)*	uses processing skills with limited effectiveness	uses processing skills with some effectiveness	uses processing skills with considerable effectiveness	uses processing skills with a high degree of effectiveness
Use of critical/creative thinking processes* *(e.g., making and testing conjectures, posing and solving problems, critiquing solutions, providing mathematical reasoning)*	uses critical/ creative thinking processes with limited effectiveness	uses critical/ creative thinking processes with some effectiveness	uses critical/ creative thinking processes with considerable effectiveness	uses critical/ creative thinking processes with a high degree of effectiveness

* The processing skills and critical/creative thinking processes in the Thinking category include some but not all aspects of the *mathematical processes* laid out in Strand A: Social-Emotional Learning Skills in Mathematics and the Mathematical Processes. Some aspects of the mathematical processes relate to the other categories of the achievement chart.

Communication – The conveying of meaning through various forms

Categories	Level 1	Level 2	Level 3	Level 4
	The student:			
Expression and organization of ideas and information *(e.g., clear expression, logical organization)* **in oral, visual, and/or written forms** *(e.g., pictorial, graphic, numeric, algebraic forms; gestures and other non-verbal forms; models)*	expresses and organizes ideas and information with limited effectiveness	expresses and organizes ideas and information with some effectiveness	expresses and organizes ideas and information with considerable effectiveness	expresses and organizes ideas and information with a high degree of effectiveness
Communication for different audiences *(e.g., peers, adults)* **and purposes** *(e.g., to generate ideas, present data, justify a solution)* **in oral, visual, and/or written forms**	communicates for different audiences and purposes with limited effectiveness	communicates for different audiences and purposes with some effectiveness	communicates for different audiences and purposes with considerable effectiveness	communicates for different audiences and purposes with a high degree of effectiveness
Use of conventions, vocabulary, and terminology of the discipline in oral, visual, and/or written forms *(e.g., terms, symbols)*	uses conventions, vocabulary, and terminology with limited effectiveness	uses conventions, vocabulary, and terminology with some effectiveness	uses conventions, vocabulary, and terminology with considerable effectiveness	uses conventions, vocabulary, and terminology with a high degree of effectiveness

Application – The use of knowledge and skills to make connections within and between various contexts

Categories	Level 1	Level 2	Level 3	Level 4
	The student:			
Application of knowledge and skills *(e.g., representations and computational strategies)* **in familiar contexts**	applies knowledge and skills in familiar contexts with limited effectiveness	applies knowledge and skills in familiar contexts with some effectiveness	applies knowledge and skills in familiar contexts with considerable effectiveness	applies knowledge and skills in familiar contexts with a high degree of effectiveness
Transfer of knowledge and skills to new contexts *(e.g., representations and computational strategies)* **to new contexts**	transfers knowledge and skills to new contexts with limited effectiveness	transfers knowledge and skills to new contexts with some effectiveness	transfers knowledge and skills to new contexts with considerable effectiveness	transfers knowledge and skills to new contexts with a high degree of effectiveness
Making connections within and between various contexts *(e.g., connections to everyday and real-life situations; connections involving an understanding of the relationships between different measurements; connections among concepts, representations, and forms within mathematics; connections involving use of prior knowledge and experience; connections among mathematics and other disciplines, including other STEM [science, technology, engineering, and mathematics] subjects)*	makes connections within and between various contexts with limited effectiveness	makes connections within and between various contexts with some effectiveness	makes connections within and between various contexts with considerable effectiveness	makes connections within and between various contexts with a high degree of effectiveness

Criteria and Descriptors for Mathematics

To guide teachers in their assessment and evaluation of student learning, the achievement chart provides "criteria" and "descriptors" within each of the four categories of knowledge and skills.

A set of criteria is identified for each category in the achievement chart. The criteria are subsets of the knowledge and skills that define the category. The criteria identify the aspects of student performance that are assessed and/or evaluated, and they serve as a guide to what teachers look for. In the mathematics curriculum, the criteria for each category are as follows:

Knowledge and Understanding

- knowledge of content (e.g., math facts, computational strategies, terminology, mathematical models, money values)
- understanding of content (e.g., concepts, theories, procedures, principles, mathematical processes)

Thinking

- use of planning skills (e.g., interpreting and expressing problems, identifying unknown(s), making conjectures and estimates, identifying steps to take, considering the use of models and representations, selecting strategies and tools)
- use of processing skills (e.g., carrying out plans: collecting data, questioning, testing, revising, modelling, solving, inferring, forming conclusions; looking back at solutions: reflecting, evaluating reasonableness, reasoning, justifying, proving)
- use of critical/creative thinking processes (e.g., making and testing conjectures, posing and solving problems, critiquing solutions, providing mathematical reasoning)

Communication

- expression and organization of ideas and information (e.g., clear expression, logical organization) in oral, visual, and/or written forms (e.g., pictorial, graphic, numeric, algebraic forms; gestures and other non-verbal forms; models)
- communication for different audiences (e.g., peers, adults) and purposes (e.g., to generate ideas, present data, justify a solution) in oral, visual, and/or written forms
- use of conventions, vocabulary, and terminology of the discipline in oral, visual, and/or written forms (e.g., terms, symbols)

Application

- application of knowledge and skills (e.g., representations and computational strategies) in familiar contexts
- transfer of knowledge and skills (e.g., representations and computational strategies) to new contexts

- making connections within and between various contexts (e.g., connections to everyday and real-life situations; connections involving an understanding of the relationships between different measurements; connections among concepts, representations, and forms within mathematics; connections involving use of prior knowledge and experience; connections among mathematics and other disciplines, including other STEM [science, technology, engineering, and mathematics] subjects)

"Descriptors" indicate the characteristics of the student's performance, with respect to a particular criterion, on which assessment or evaluation is focused. *Effectiveness* is the descriptor used for each of the criteria in the Thinking, Communication, and Application categories. What constitutes effectiveness in any given performance task will vary with the particular criterion being considered. Assessment of effectiveness may therefore focus on a quality such as appropriateness, clarity, accuracy, precision, logic, relevance, significance, fluency, flexibility, depth, or breadth, as appropriate for the particular criterion.