

$$\begin{aligned}
& + \frac{\partial^{\alpha_2-1}}{\partial w^{\alpha_2-1}} \left[ \int_0^w \frac{\partial}{\partial w} \frac{\partial^{\alpha_1}}{\partial z^{\alpha_1}} g(z, w-v) \frac{\partial}{\partial v} f(0, v) dv \right] \\
& + \left( \frac{\partial^{\alpha_1}}{\partial z^{\alpha_1}} g(z, 0) \left( \frac{\partial}{\partial w} f \right)(0, w) \right) \\
& + f(0, 0) \frac{\partial^{|\alpha|}}{\partial z^{\alpha_1} \partial w^{\alpha_2}} g(z, w) \\
& = I_1 + I_2 + I_3 + \frac{\partial^{\alpha_1-2}}{\partial z^{\alpha_1-2}} \left[ \int_0^{\frac{\partial}{\partial z} f} \int_0^2 \frac{\partial}{\partial z} \frac{\partial^{\alpha_2}}{\partial w^{\alpha_2}} g(z-u, w) \frac{\partial}{\partial u} f(u, 0) du \right. \\
& \quad \left. + \left( \frac{\partial^{\alpha_2}}{\partial w^{\alpha_2}} f(0, w) \right) \left( \frac{\partial^2}{\partial z^2} f(z, 0) \right) \right] \\
& + \frac{\partial^{\alpha_2-2}}{\partial w^{\alpha_2-2}} \left[ \int_0^w \frac{\partial}{\partial w} \frac{\partial^{\alpha_1}}{\partial z^{\alpha_1}} g(z, w-v) \frac{\partial}{\partial v} f(0, v) dv \right] \\
& + f(0, 0) \frac{\partial^{|\alpha|}}{\partial z^{\alpha_1} \partial w^{\alpha_2}} g(z, w) \\
& = I_1 + I_2 + I_3 + \\
& + \frac{\partial^{\alpha_1-2}}{\partial z^{\alpha_1-2}} \left[ \int_0^{\frac{\partial}{\partial z} f} \int_0^2 \frac{\partial^2}{\partial z^2} \frac{\partial^{\alpha_2}}{\partial w^{\alpha_2}} g(z-u, w) \frac{\partial}{\partial u} f(u, 0) du \right. \\
& \quad \left. + \left( \frac{\partial}{\partial z} \frac{\partial^{\alpha_2}}{\partial w^{\alpha_2}} f(0, w) \right) \left( \frac{\partial}{\partial z} f(z, 0) \right) + \frac{\partial^{\alpha_2}}{\partial w^{\alpha_2}} g(0, w) \frac{\partial^2}{\partial z^2} f(z, 0) \right] \\
& + \frac{\partial^{\alpha_2-2}}{\partial w^{\alpha_2-2}} \left[ \int_0^w \frac{\partial^2}{\partial w^2} \frac{\partial^{\alpha_1}}{\partial z^{\alpha_1}} g(z, w-v) \frac{\partial}{\partial v} f(0, v) dv \right]
\end{aligned}$$

$$\begin{aligned}
& + \left[ \int_0^w \frac{\partial^{\alpha_1+\alpha_2}}{\partial w^{\alpha_2} \partial z^{\alpha_1}} g(z, w-v) \frac{\partial}{\partial v} f(0, v) dv + \left( \frac{\partial^{\alpha_2-1}}{\partial w^{\alpha_2-1}} g(z, 0) \frac{\partial}{\partial w} f(0, w) \right) \right. \\
& \quad \left. + \frac{\partial^{\alpha_1}}{\partial z^{\alpha_1}} g(z, 0) \frac{\partial^{\alpha_2}}{\partial w^{\alpha_2}} f(0, w) \right] \\
& + f(0, 0) \frac{\partial^{|\alpha|}}{\partial z^{\alpha_1} \partial w^{\alpha_2}} g(z, w).
\end{aligned}$$

So, we obtain that

$$\begin{aligned}
& \frac{\partial^{|\alpha|}}{\partial z^{\alpha_1} \partial w^{\alpha_2}} (f \otimes g)(z, w) \\
& = \int_0^z \int_0^w \frac{\partial^{|\alpha|}}{\partial z^{\alpha_1} \partial w^{\alpha_2}} g(z-u, w-v) \frac{\partial^2}{\partial u \partial v} f(u, v) dv du \\
& + \int_0^z \frac{\partial^{\alpha_1}}{\partial z^{\alpha_1}} g(z-u, 0) \frac{\partial^2}{\partial u \partial w} f(u, w) du \\
& + \int_0^w \frac{\partial^{\alpha_2}}{\partial w^{\alpha_2}} g(0, w-v) \frac{\partial^2}{\partial z \partial v} f(z, v) dv \\
& + \int_0^z \int_0^w \frac{\partial^{|\alpha|}}{\partial z^{\alpha_1} \partial w^{\alpha_2}} g(z-u, w) \frac{\partial}{\partial u} f(u, 0) du \\
& + \left( \frac{\partial^{|\alpha|-1}}{\partial z^{\alpha_1-1} \partial w^{\alpha_2}} g \right)(0, w) \frac{\partial}{\partial z} f(z, 0) + \frac{\partial^{\alpha_2}}{\partial w^{\alpha_2}} g(0, w) \frac{\partial^{\alpha_1}}{\partial z^{\alpha_1}} f(z, 0)
\end{aligned}$$



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