

Lewis Carroll syllogism from Lecture #2

- i) Every one who is sane can do Logic.
- ii) No lunatics are fit to serve on a jury.
- iii) None of *your* sons can do logic.

A first step is to rewrite these statements as 'if-then' statements:

- i) If he is sane, then he can do logic.
- ii) If he is a lunatic (not sane), then he is not fit for jury duty.
- iii) If he is your son, then he cannot do logic.

We can then write these statements symbolically. This is sometimes helpful. Let...

- $Sa = \text{'he is sane'}$ and $\text{not } Sa = \text{'he is not sane (lunatic)'}$
- $L = \text{'he can do logic'}$ and $\text{not } L = \text{'he cannot do logic'}$
- $J = \text{'he is fit for jury duty'}$ and $\text{not } J = \text{'he is not fit for jury duty'}$
- $So = \text{'he is your son'}$ and $\text{not } So = \text{'he is not your son.'}$

The above implications can then be written symbolically:

- i) $Sa \rightarrow L$ which has contrapositive $\text{not } L \rightarrow \text{not } Sa$
- ii) $\text{not } Sa \rightarrow \text{not } J$ which has contrapositive $J \rightarrow Sa$
- iii) $So \rightarrow \text{not } L$ which has contrapositive $L \rightarrow \text{not } So$

Recall, the contrapositive is logically equivalent the original implication. That means if we assume our original implications are TRUE, then all of the contrapositives are TRUE. We can piece together these statements (written symbolically):

$$J \rightarrow Sa \text{ and } Sa \rightarrow L \text{ and } L \rightarrow \text{not } So$$

- If he is fit for Jury, then he is sane. [Contrapositive of ii)]
- If he is sane, then he can do logic. [implication i)]
- If he can do logic, then he is not your son [contrapositive of iii)]

Piecing these together, we can say $J \rightarrow \text{not } So$ or rewritten

If he is fit for jury duty, then he is not your son ... or ... None of your sons are fit for jury duty.

Likewise, we could piece together:

$$So \rightarrow \text{not } L \text{ and } \text{not } L \rightarrow \text{not } Sa \text{ and } \text{not } Sa \rightarrow \text{not } J$$

- If he is your son, then he cannot do logic. [implication iii)]
- If he cannot do logic, then he is not sane. [contrapositive of i)]
- If he is not sane, then he is not fit for Jury. [implication ii)]

Piecing these together, we can say $So \rightarrow \text{not } J$ or rewritten (contrapositive of above)

If he is your son, then he is not fit for jury duty ... or ... your sons are not fit for jury duty.