THE BOLTS AND COMPRESSED PLATES MODELLING

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Keywords: Steel structures, bolted connection, bolts in tension and shear, triangular stiffener, finite element method, reduced stress method.

Abstract. *The paper is focused on modelling of the bolts in tension and plate in bending and of compressed plate in structural steel connection by analytical and finite element models. The procedure for creating a Design finite element model (DFEM) with shell elements and a Component based finite element model (CBFEM) with shell elements and component elements for bolts, weld, and compressed plate, is discussed. The proposed procedure is evaluated on published and performed experiments.*

1   INTRODUCTION

End-plate joints are due to their simplicity and cost one of the most common connection type in steel structures. End-plate joints are usually designed using analytical component based method (CBM). This method allows relatively precise design of joints with usual geometry. Designed joint must contain only known components which are described in the design codes or literature. Another limitation of the CBM can be faced when the joint is loaded by combinations of internal forces, as some combinations (for example combination of strong and weak axis bending moments or combination with the axial force) are not supported. Silva (2008) [1] creates a matrix that takes into account the influence of a combination of internal forces. However, this issue of interaction is inaccurate and complex form its principle and its approval is just at the beginning.

Design finite element model (DFEM) and Component based finite element model (CBFEM) are getting common for design of joints as an alternative solution. Application of Research finite element model (RFEM) is time-consuming. Introduction of non-linearity of the bolts, welds or compressed plates exhibiting stability problems bring difficulties. Prediction of failure of individual parts of the joint is unclear because of stress peaks. Despite these problems, many numerical models have already been created in the past [2]. These models were usually used for parametric studies and were based on the results of experiments. Results of these studies are useful, however these studies did not consider the possibility of using finite element method for everyday design of joints to make it available as an alternative approach to the design of joints using component method. This paper examines numerical models of bolts and compressed plates as first step to create the Component based model (CBM) and the Design finite element model (DFEM) or Component based finite element model (CBFEM) of end plate joint.

2   BOLT IN TENSION AND PLATE IN BENDING

2.1 Bolts in tension

Several analytical models of bolts in tension have been developed. These analytical models consider linear behaviour with initial stiffness up to bearing capacity of the bolt. The most commonly used procedure is the T stub model published in chapter 6 of EN1993-1-8:2008. For bolt elongation

developed Agerskov (1976) [3] an analytical model that considers the deformation of the shank and thread, nut and washers according to equations (1), (2), and (3), respectively. Initial stiffness is determined from the sum of deformations of these parts.

$$\Delta l_{shank} = \Delta l_s + \Delta l_t = \frac{N_t}{EA_s}l_s + \frac{N_t}{EA_t}\left(l_t + \frac{l_n}{2}\right) \tag{1}$$

$$\Delta l_n = \frac{N_t}{2EA_n}l_n \tag{2}$$

$$\Delta l_w = \frac{N_t}{EA_w}l_w \tag{3}$$

where subscripts n, s, t, w refer to nut shaft, thread and washer, respectively.

Similar approach is used in the analytical model according to the German guideline VDI2230 [4] equations (4), (5), and (6) and according to Barron and Bickford (1998) [5] equation (7). These two models consider deformation caused by stripping of the threads in thread-nut contact area in addition.

$$\Delta l_b = \Delta l_{b1} + \Delta l_{b2} = \frac{N_t}{EA_s}(l_s + 0{,}4d_b) + \frac{N_t}{EA_t}(l_t + 0{,}85d_b) \tag{4}$$

$$\Delta l_w = \frac{N_t}{EA_p}l_w \tag{5}$$

where $$A_p = \frac{\pi}{4}(d_h^2 - d_{w1}^2) + \frac{1}{2}(d_{w2}^2 - d_h^2)tan^{-1}\left[\frac{0{,}75d_h(l_w - d_h)}{(d_{w2}^2 - d_{w1}^2)}\right] \tag{6}$$

in which d_b, d_h, d_{w1} and d_{w2} denotes bolt shank diameter, head diameter, washers inner and outer diameters, respectively. Stiffness is determined from the sum of deformations Δl_b and Δl_w.

$$\frac{1}{K_b} = \frac{fd_b}{EA_s} + \frac{l_s}{EA_s} + \frac{l_t}{EA_t} + \frac{fd_b}{EA_t} \tag{7}$$

where f is a correlation factor determined by Swanson [6] as the average value 0,73.

Simplified procedure is used in Eurocode 3 when constant cross-section corresponding to the core is used along entire length of the bolt which is taken as the clamping length. The stiffness calculated from these data is then reduced to 80%, see equation (8).

$$K = \frac{0{,}8EA_t}{l_b} \tag{8}$$

where $$l_b = l_s + l_t + \frac{l_h + l_n}{2} \tag{9}$$

and l_h denotes length of the bolt head.

In addition to these analytical models many numerical models of the bolts have been created. Many approaches that use different element types were applied to numerical models of the bolts in the past. Tarpy and Cardinal (1981) [7] created 2-D models and used shell elements for bolts. 3-D models started to be used when powerful computers were available. In these models, spring elements were used by Bahaari and Sherbourne (1996) [8] and beam elements by Bursi and Jaspart (1998).

Later, solid elements were applied to model of bolts. However, some simplifications were necessary in numerical models because of the time and computational limits. Kukretti and Zhou (2006) [9] created a numerical model of the bolts, which neglected the washers and considered a constant nominal diameter of the bolt shank along the entire length. Chen and Du (2007) [10] neglected washers in its model too and considered a constant effective diameter of the shanks along the entire length. Wheeler et al. (2000) [11] developed a model that considers the nominal diameter in the shank and effective diameter in the threaded part. Washers are usually introduced as increase of the height of the bolt head and nut. The length of the shank or the thread corresponds to total thickness of steel plates in all three above mentioned models. Gantes et al. (2003) [12] considered effective length of the shaft according to Agerskov's model (1976). This model takes into account the deformation of the nut and thread. Washers in this model are considered by increasing the height of the bolt head and nut.

Wu et al. (2012) [13] tried to create the accurate model of the bolt, in which the separate washers and the thread are modelled. This model cannot be used in the T-stub or the joint model due to its complexity. The results were compared to the results of the four above-mentioned simplified models. Based on this comparison, new simplified model based on the model by

Wheeler et al. (2000) was created. The washers were introduced as increased height of the bolt head and nut but were separated from the bolt shank in this model. This modification takes into account the correct length of the bolt shank and the model exhibits more accurate behaviour.

2.2 Finite element model of initial stiffness

Numerical model is created in Midas FEA software. A solid numerical model of the bolt was created first and spring model was created based on the results of solid model. Spring model is much easier to create and less time consuming. Both bolts models were integrated into the T-stub model and the results of both models were compared.

Solid numerical model is based on the model by Wu et al. (2012). Nominal diameter is considered in the shank and effective core diameter is considered in the threaded part. Washers are coupled with bolts head and nut. Deformation caused by stripping of the threads in thread-nut contact area is simply modelled using interface elements. These interface elements are unable to transfer tensile stresses. Constant shear modulus was determinate to 7000N/mm^3 and normal stiffness modulus in compression is determinate as 10E/a where E is Young's modulus and is the element size. Numerical model was calibrated according to results of Wu et al. (2012) model and several analytical models.
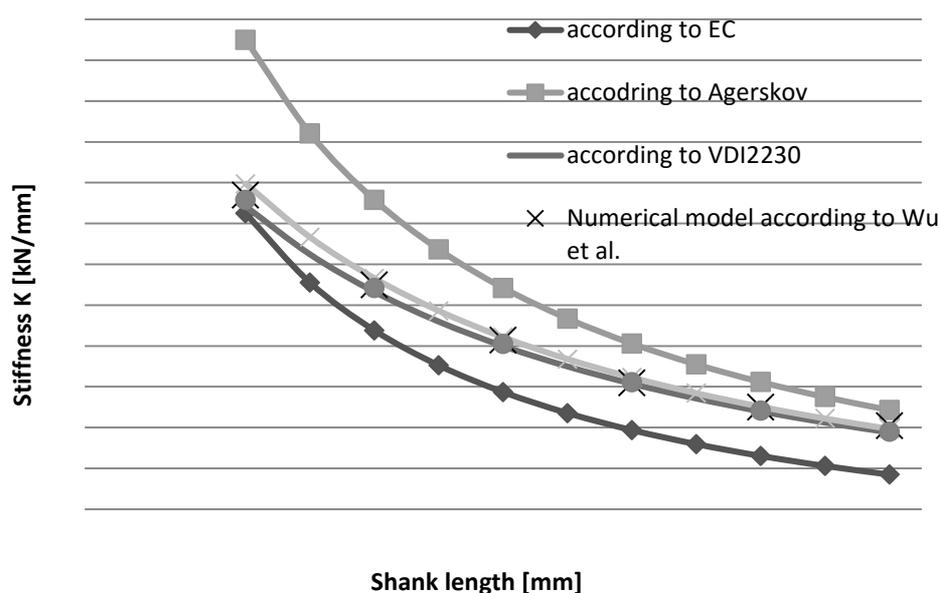

Figure 1: Parametric study of bolts initial stiffness, parameter shank length

Calibration of numerical model was done on bolt M20 with thread length of 10 mm and shank length as variable parameter. Comparison of the results is shown in Figure 1. Developed numerical model shows good agreement with model by Wu et al. Analytical model according to the VDI2230 fits good to results of numerical models and it is used for the other parametric studies of developed model. Parametric study for bolt M20 with shank length 20 mm was done. Thread length was a parameter in this case. Initial stiffness of developed numerical model was compared to analytical models because there were no data of numerical model by Wu et al. for this case of study. Results of parametric study of thread length are shown in Figure 2.

The same parametric studies were done for bolts M16, M24, M27 and M30. Initial stiffness calculated by the developed numerical model fits to results of analytical model according to the German VDI2230 very good in all cases.
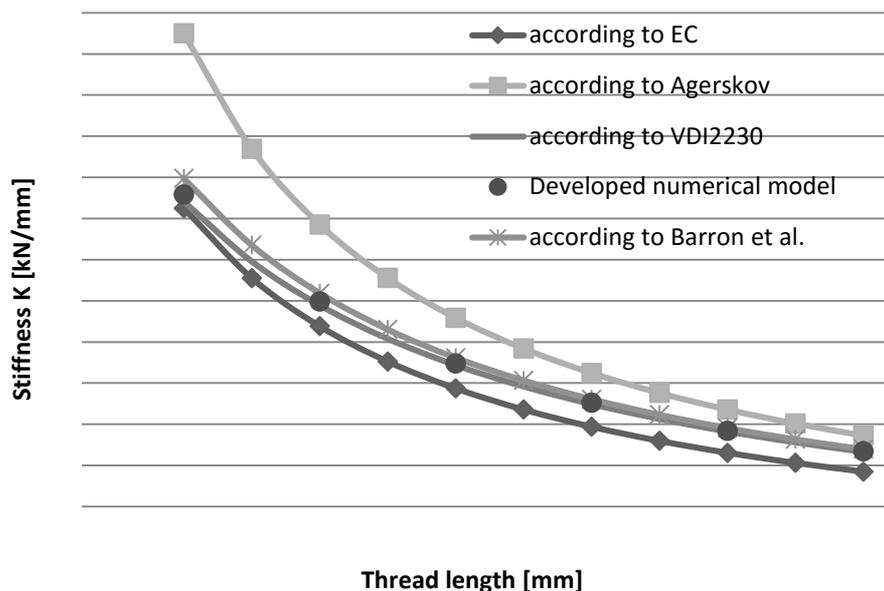
Figure 2: Parametric study of bolts initial stiffness, parameter thread length

2.3 Force-deformation diagram

Bilinear stress-strain diagrams were used in the second step. Bolt grades 4.6, 5.6, 8.8 and 10.9 were investigated. Characteristic values for yield stress f_y and ultimate strength f_u were considered. Maximal plastic strain is considered 5%. Some bolts shows bilinear or in some cases trilinear behaviour up to design tensile resistance according to EC. Such behaviour shows bolts of material with ratio f_u/f_y higher than $\gamma_{M2}/k_2 = 1{,}25/0{,}9 = 1{,}39$. Above mentioned analytical models consider only linear behaviour of bolt. Therefore, analytical model according to the VDI2230 was extended. Force-displacement diagram of bolt according to extended analytical model is shown on Figure 3. Initial stiffness E_{ini} is considered according to the VDI2230. Forces $F_{y,t}$, $F_{y,s}$, and $F_{u,t}$, correspond to initialization of thread yielding, shank yielding and threat rupture, respectively. Plastic deformations of the thread $\delta_{pl,t}$ and shank $\delta_{pl,s}$ are given by Eq. (10), (11). Shank yielding occurs only in some cases.

$$\delta_{pl,t} = \frac{F - F_{y,t}}{E_{pl}A_t}(l_t - cd_t) \tag{10}$$

$$\delta_{pl,s} = \frac{F - F_{y,s}}{E_{pl}A_s}l_s \tag{11}$$

In the above formulas, A_t, A_s, l_t, l_s, and d_t denotes thread cross section area, shank cross section area, length of thread, length of shank and threaded part diameter, respectively. Correlation factor c considers unequal stresses distribution in the threaded part shown in Figure 4. Its value depends on bolt grade, l_s/l_t ratio and is in range from 0,3 to 0,6. Plastic modulus E_{pl} is given by equation (12).

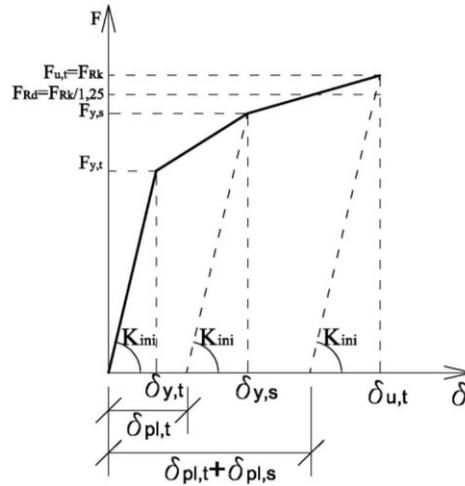$$E_{pl} = \frac{f_u - f_y}{\varepsilon_{pl}} \tag{11}$$

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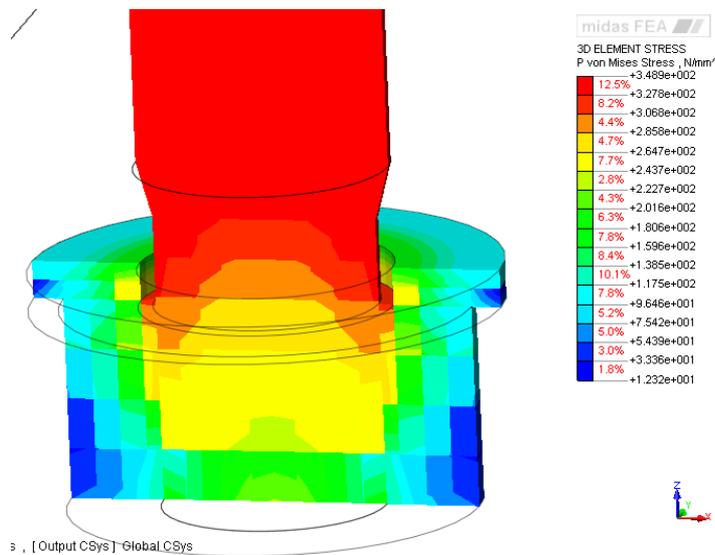Figure 3: Force-displacement diagram according to developed analytical model

Figure 4: Unequal stresses distribution in the threaded part of the bolt

Comparison of force-displacement diagrams is shown in Figure 5. This comparison is made for two bolts M24 grade 8.8 with length of the shank 50 mm. The graph on the left shows the force-displacement diagram for bolt with thread length of 17,6 mm and the graph on the right shows force-displacement diagram for bolt with thread length of 28,2 mm.
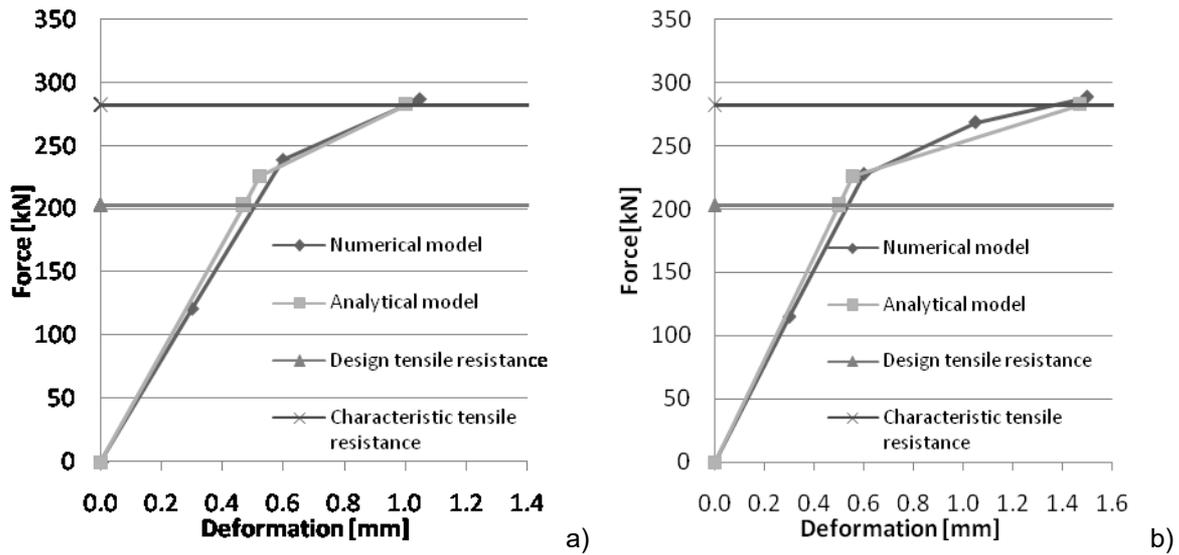
Figure 5: Force-displacement diagrams for bolts M24 grade 8.8. Thread length 17,6 mm a), thread length 28,2 mm b).

2.4 T-stub

The accuracy of the bolts behaviour was subsequently validated on experiments of two T-stubs performed at the CTU Prague. T-stubs were made from hot-rolled sections HEB300 and HEB400 and were connected by two bolts M24 8.8. Two different numerical models have been created. The first numerical model considers a detailed solid model of the bolt, shown in Figure 6a. One quarter of the sample was modelled for the reason of symmetry. Comparison of results of the numerical models to experimental results for both experiments are shown on graphs in Figure 7.
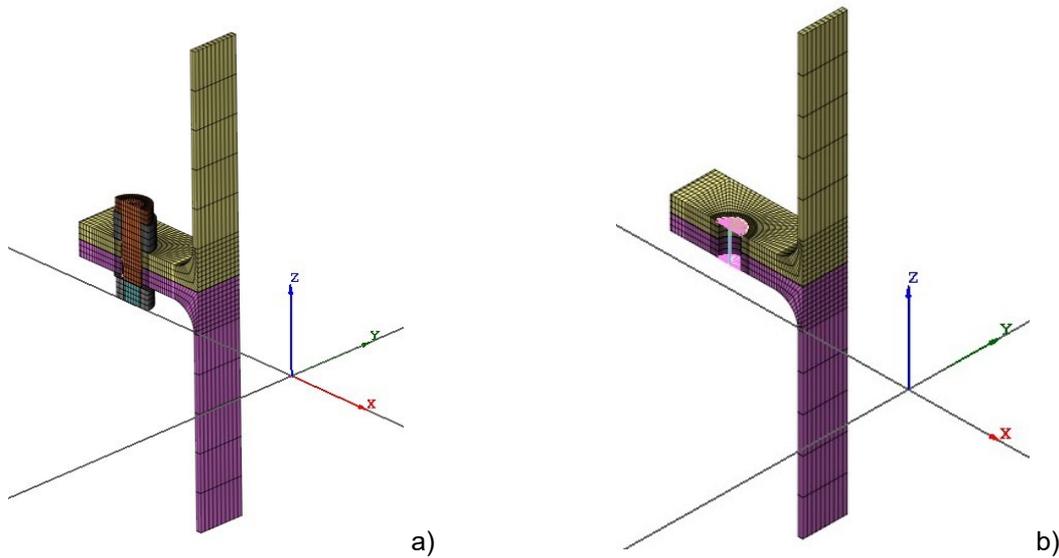

Figure 6: T-stub model a) with solid element bolts, b) with spring element bolts
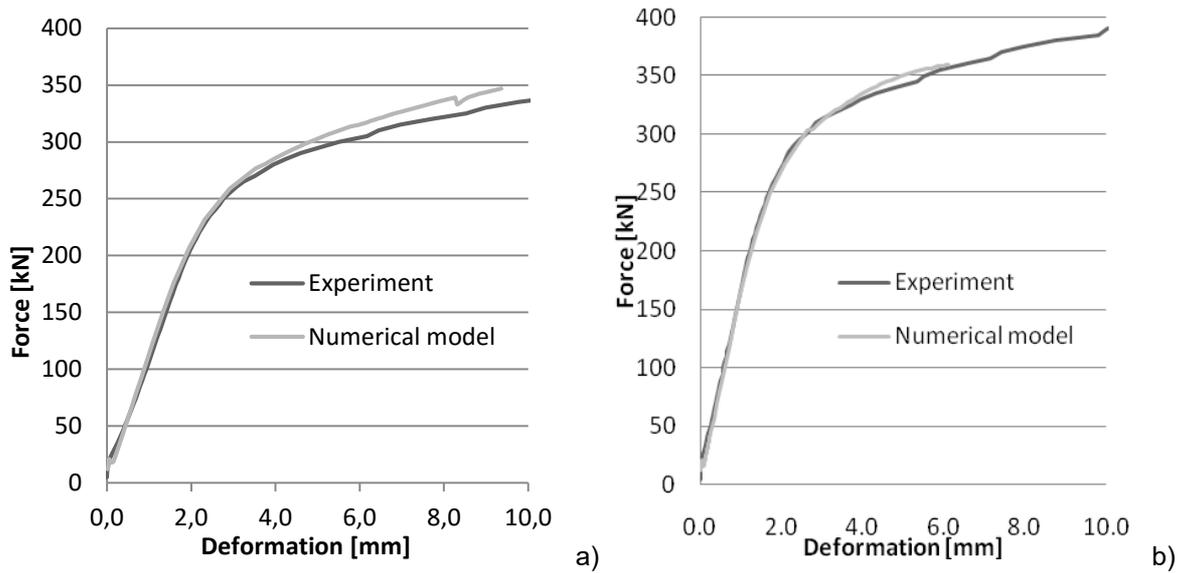
Figure 7: Force-deformation diagrams for T-stubs, HEB300 sample a), HEB400 sample b).

The spring elements are used for the bolts in second model, see Figure 6b. Rigid elements are used on the ends of spring to transmit forces into the flange of the T-stub. Bilinear stress-strain diagram according to the proposed analytical model has been set for spring elements. Connection point of rigid elements to the flange significantly affects the behaviour of T-stub. The graphs in Figure 8 show influence of the radius of the rigid elements. Three different radii of rigid elements were chosen. The first corresponds to bolt-hole radius, the second corresponds to outer radius of the bolt head and the third corresponds to middle radius of the bolt head. It is clear that the radius of the rigid elements corresponds to middle radius of the bolt head shows the best agreement with experiment. Model with spring element bolts shows a slightly smaller stiffness compared to model with solid element bolts, see Figure 9. This is due to omitted bending stiffness of the spring elements.
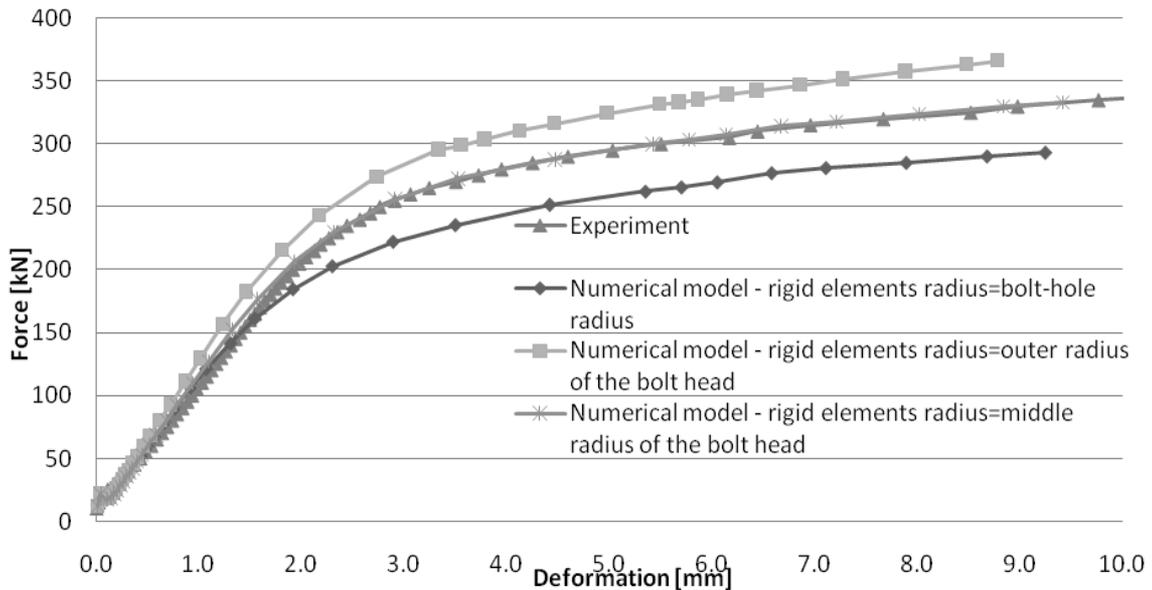

Figure 8: Influence of the radius of the rigid elements to T-stub deformation
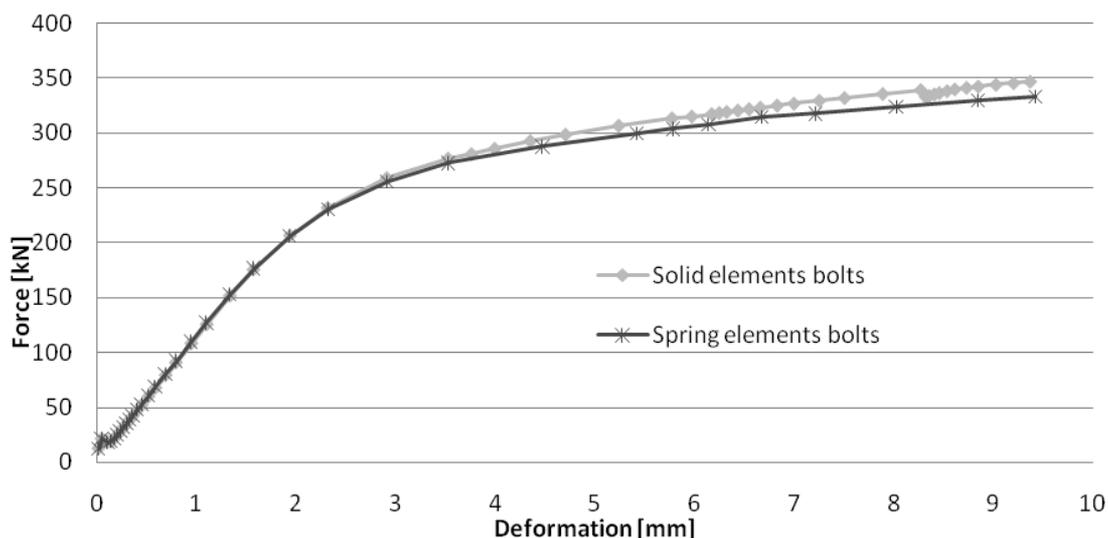
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Figure 9: Comparison of solid elements bolt model and spring elements bolt model

4   COMPRESSED PLATES

4.1 Models

Generally the design of compressed plates could be solved by two approaches. The first one is based on geometric and material nonlinear analysis with imperfections. Modelling of imperfections in complex connections may be complicated task and the analysis is time-consuming. The second approach is based on material nonlinear and geometric linear analysis without imperfections and manual verification of compressed slender plates. For the verification is proposed to use reduced stress method provided in EN 1993-1-5 [14]. The verification is based on the von-Mises yield criterion and sums up the load effects of normal and shear stresses

$$\left(\frac{\sigma_{x,Ed}}{\rho_x f_y/\gamma_{M1}}\right)^2 + \left(\frac{\sigma_{z,Ed}}{\rho_z f_y/\gamma_{M1}}\right)^2 - \left(\frac{\sigma_{x,Ed}}{\rho_x f_y/\gamma_{M1}}\right)\left(\frac{\sigma_{z,Ed}}{\rho_z f_y/\gamma_{M1}}\right) + 3\left(\frac{\tau_{Ed}}{\chi_w f_y/\gamma_{M1}}\right)^2 \leq 1 \qquad (13)$$

4.2 Verification

The verification procedure is divided in four steps. In first step is performed the material nonlinear analysis without imperfections of the connection model. In second step is a compressed plate separated from the connection and the boundary condition and stress distribution are determined. In the next step is carried out linear buckling analysis to obtain the first eigenmode and critical buckling factor of the separated plate. In the last step are determined reduction factors ρ_x, ρ_z and χ_w for longitudinal, transverse and shear stress and the plate is verified using reduced stress method.

An example of numerical model with triangular stiffener in beam-column connection is shown in Figure 10. The numerical model is created in Dlubal RFEM software, using shell elements and bilinear material model with strain hardening. The plasticization in stiffener is significant, although it is not clear, if the ultimate resistance is reached.
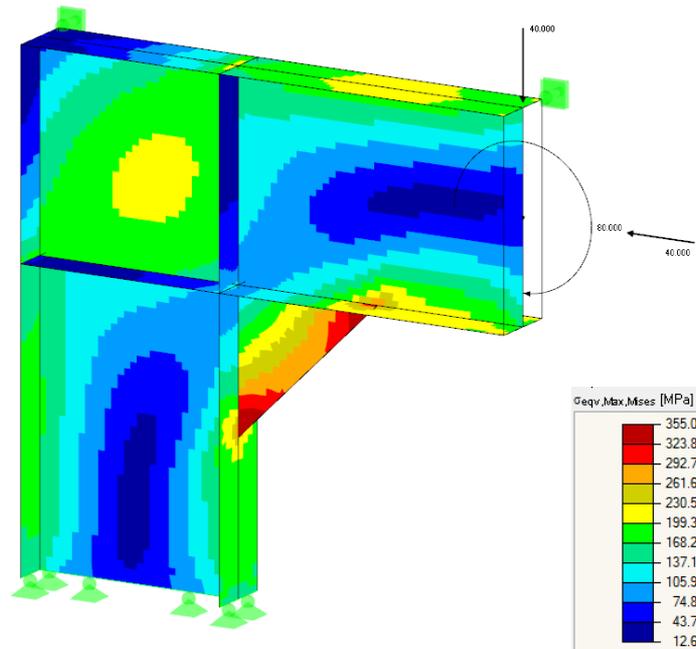
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Figure 10: Von-Mises stress distribution for beam-to-column joint

The triangular stiffener is taken out of the model, boundary conditions are chosen CCF (clamped, clamped, free edge) and the stress distribution is applied, as shown in Figure 11a).In Midas FEA software are for the given boundary conditions and stress distribution calculated first eigenmode and critical buckling factor as shown in Figure 11b).
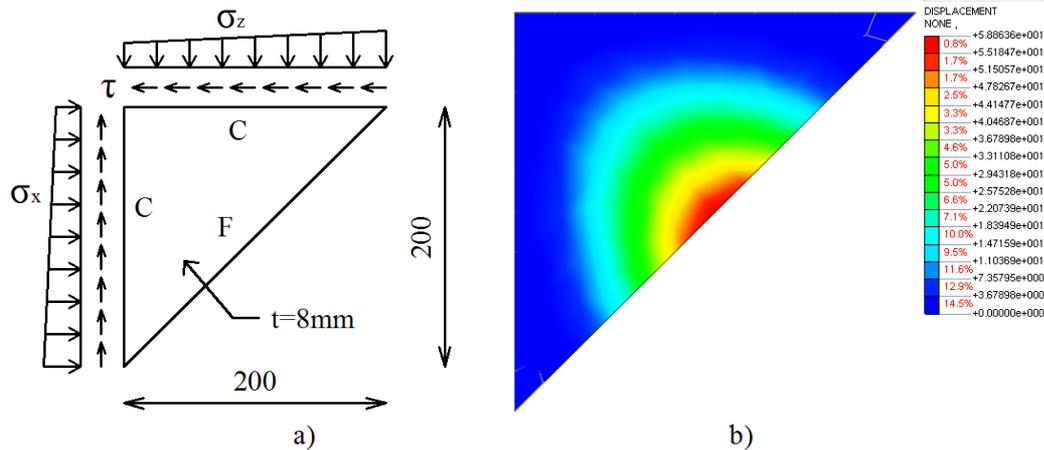

Figure 11: Triangular stiffener
a) boundary conditions and stress distribution b) first eigenmode

Reduction factors for longitudinal, transverse and shear stress may be determined from buckling curves according to EN 1993-1-5 or using geometric and material nonlinear analysis. Buckling curves are verified for square or rectangular plates but not for non-regular shapes as triangular. For this example are reduction factors determined in Midas FEA. Finally the sum of the load effects shows that the verification is not satisfied and ultimate strength is overstepped.

5   CONCLUSIONS

The current analytical models consider only linear behaviour of the bolts. This simplification may be inaccurate, especially for bolts of materials with the f_u/f_y ratio higher than $\gamma_{M2}/k_2 = 1{,}25/0{,}9 = 1{,}39$ for the Design finite element model (DFEM) or Component based finite element model (CBFEM). Analytical model according to the VDI2230 was extended to plastic deformations. Detailed numerical model can be used for an accurate description of the behaviour

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of the bolts. Numerical model, based on the model by Wu et al. (2012) was created. Developed numerical model differs by using interface elements to consider deformation caused by stripping of the thread in the thread-nut contact area. The proposed numerical model is much less time consuming. Despite the simplification, a detailed numerical model is still very time consuming and cannot be used for everyday design, therefore spring model of the bolt was created. Rigid elements are used at the ends of the spring elements to transmit forces into the flange. The spring model was validated by the results of T-stub experiments. The Research finite element model (RFEM)of T-stub with spring element bolts shows slightly lower stiffness because of neglected bending stiffness. Differences in stiffness are negligible and can be ignored. The spring element bolt is time-saving and can be advantageously used for the design by CBM, DFEM, and CBFEM of end-plate joints.

It is proposed to use the reduced stress method for the Design finite element model (DFEM) or Component based finite element model (CBFEM) of compressed plates in connections. The verification example shows that compressed plates could be in finite element models designed without applying imperfections. Essential part of the following research is verification of buckling curve for non-regular plate shapes such as triangular. Using the buckling curve instead of nonlinear analysis will reduce the calculation time. The procedure will be validated on the published and newly prepared experiments of stiffeners with different types of support, free edge, partially stiffened and clamped.

ANNOUNCEMENT

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